The Bang for the Buck:

Aggregate Impact of Firm-Level R&D Incentives

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Abstract

How do different firms respond to R&D incentives and, in turn, shape aggregate growth? We develop a novel empirical framework, grounded in endogenous growth theory, allowing us to measure firms' responsiveness to R&D incentives and to aggregate such responses. After validating the predictions of our framework using several micro-datasets, we apply it to Compustat data. We find that (i) ignoring firm heterogeneity severely under-states the aggregate effectiveness of R&D incentives, (ii) per dollar spent on R&D incentives, young (rather than small) firms raise aggregate growth the most, and (iii) our results are robust to knowledge spillovers, dynamics, and borrowing constraints.

1 Introduction

With global economic growth forecasted to hit historical lows (see, e.g., World Bank, 2025), there is renewed interest in industrial policies aimed at promoting research and development (R&D). Despite many countries using various tax incentives and subsidies to spur growth—often targeted at specific groups of firms such as small businesses (see, e.g., Bloom et al., 2019; OECD, 2023)—little consensus exists about how such policies affect different firms and how these differences may impact aggregate growth.

This paper examines which groups of incumbent firms generate the greatest "Bang for the Buck"—a boost to aggregate growth per dollar spent on R&D incentives. We develop a novel empirical framework which allows us to (i) measure how *individual firms* respond to R&D incentives using a directly observable sufficient statistic and (ii) aggregate these firm-level responses to estimate the overall impact on *aggregate* growth. While our framework is grounded in modern endogenous growth theory, it does not require us to solve complex models with heterogeneous firms. Instead, our sufficient statistics approach allows us to directly incorporate the full extent of granular differences across firms observed in the data.

Before applying our framework, we validate our predictions in several micro-datasets. We do so by showing that our sufficient statistics closely align with existing estimates of average responses of firms to R&D incentives based on difference-in-differences approaches (see, e.g., Bloom et al., 2002; Hall et al., 2010; Appelt et al., 2025). In addition, using firm-level administrative data, we provide novel causal evidence that firms' responsiveness to R&D incentives moves one-for-one with our sufficient statistics.

Applying our framework to Compustat data, three key messages stand out. First, firm heterogeneity matters. The inability to measure responsiveness to R&D policies at the firm-level—the typical case in the existing literature—under-estimates the impact such policies have on aggregate growth by a factor of seven. This is because, in the data, firms that are relatively responsive to R&D incentives also tend to be relatively large and fast growing, strengthening the Bang for the Buck. Second, we show that young and fast-growing businesses deliver the largest Bang for the Buck. While this contrasts with existing policies that often target small businesses, it aligns well with the established economic prowess of young firms (see, e.g., Haltiwanger et al., 2016). We contribute to this discussion by showing that young firms are also important propagators of R&D policies. Finally, we document that our results do not change even when accounting for knowledge spillovers across businesses, dynamics, or borrowing constraints.

Our analytical approach rests on two ingredients. First, we consider an environment in which aggregate output is produced by potentially heterogeneous firms. Therefore, aggregate economic growth is a weighted average of firm-level growth rates with firms' sales shares representing the (Domar) weights. Second, we assume that, in order to grow, firms invest into R&D by optimally balancing associated marginal costs and benefits. While the former are governed by firms' cost structures, the latter are driven by changes in firms' profits (firm value) stemming from successful innovation.

In this environment, we consider a permanent increase in R&D subsidies and define the following objects. The "Bang" is the associated impact effect on aggregate growth. The "Buck" is the associated impact effect on aggregate government spending on R&D incentives. The "Bang for the Buck" is the ratio of the two, measuring how aggregate growth changes on impact per extra dollar spent on the policy change. Given that aggregates in our framework are driven by individual firms, a crucial part of our analysis focuses on how individual businesses respond to changes in R&D incentives.

Instead of relying on policy differences (across space or time) to estimate firms' responsiveness, we lean on the fundamental economic tradeoffs inherent to firms' optimal R&D decisions. Specifically, as R&D becomes cheaper due to more generous innovation incentives, firms optimally increase their R&D investment. On the one hand, this comes at higher costs. On the other hand, it increases expected growth and, therefore, future profits. How exactly firms balance these (potentially idiosyncratic) trade-offs informs us about their overall responsiveness ("micro-elasticity"). We show analytically that, under a certain condition, firm-level micro-elasticities are fully described by observable sufficient statistics: firms' ratios of R&D expenditures to profits—the two sides of firms' R&D decisions.

The condition behind the above result is that R&D to profit ratios are constant over firms' life-cycles (though potentially heterogeneous across businesses). We validate this condition and the implied micro-elasticities in three different ways. First, we discuss that this condition does in fact hold in a very wide range of existing endogenous growth models.[1] Second, we show that R&D-to-profit ratios are indeed constant at the firm-level in Compustat data. There is, however, vast heterogeneity in this ratio *across* firms. Third, we use two additional micro-datasets and employ established difference-in-differences (DiD) approaches to estimate micro-elasticities for broad groups of firms.[2] The results show that our sufficient statistics do a good job of capturing firms' responsiveness to R&D policy changes—both on average and across different firm groups. Importantly,

[1]Examples of such models include, e.g., Klette and Kortum (2004); Luttmer (2007); Lentz and Mortensen (2008); Luttmer (2010); Mukoyama and Osotimehin (2019). The reason why this condition holds is so called "perfect scaling" whereby sales and costs grow one-for-one with firm size (see Akcigit and Kerr, 2018, for a discussion and an example of a framework which deviates from perfect scaling). In Appendix F.9, we show that if R&D to profits are not constant at the firm-level, our sufficient statistics generalize to the ratio of the net present value of future R&D expenditures and firm values. Using estimates of these generalized sufficient statistics instead does not alter our results. Moreover, in Appendix E, we consider the model in Ignaszak and Sedláček (2025)—which deviates from perfect scaling—and show that even in this model our micro-elasticities remain to proxy firms' true responsiveness to R&D incentives well.

[2]The two datasets are ORBIS, where we follow the cross-country analysis of Appelt et al. (2025), and BLADE—firm-level data from Australia, which implemented a large R&D policy change in 2012.

we provide novel direct empirical evidence that micro-elasticities—estimated using modern DiD methods—indeed move one-for-one with firms' R&D-to-profits ratios, validating a key prediction of our framework.

Having established the validity of our micro-elasticities, we use our framework to understand key trade-offs in the design of R&D subsidies. On the one hand, we show that the Bang is determined by an interaction of firms' sales shares, their growth rates, and their micro-elasticities. Intuitively, individual firms can only leave a mark on aggregate growth if they are sufficiently large, fast-growing, and responsive to R&D policy changes. On the other hand, the Buck is driven by the interaction of firms micro-elasticities and R&D shares. Intuitively, an R&D policy change will be particularly expensive for the government if firms respond to it very strongly, or if the targeted businesses already receive a substantial portion of government resources.

Therefore, our analytical framework demonstrates that understanding the aggregate impact of R&D incentives requires more than micro-elasticities alone. At the same time, not being able to measure micro-elasticities at the firm-level—a limitation common in the existing literature—provides a distorted view of the aggregate effectiveness of R&D incentives. To highlight these points, we apply our framework to Compustat, a firm-level dataset extensively used in the literature to study R&D, firm-level and aggregate growth (see, e.g., Cavenaile et al., 2021; Ignaszak and Sedláček, 2025). Three key messages emerge.

First, firm heterogeneity matters. In the data, firms characterized by higher micro-elasticities tend to be larger and grow faster. This covariance between the key firm characteristics boosts the aggregate Bang. Quantitatively, ignoring such heterogeneity understates the aggregate Bang by a factor of seven. This means that the aggregate impact of firm-level R&D incentives is potentially much larger than previously thought.[3]

Second, we turn our attention to the *relative* Bang for the Buck for various groups of firms. This statistic measures how much more (or less) Bang for the Buck the government could obtain if it were to subsidize only a specific group of firms relative to indiscriminately handing out R&D subsidies to all businesses. Our results suggest that young and fast-growing businesses offer the highest relative Bang for the Buck—almost three times as much compared to a uniform subsidy for all firms. This finding aligns well with existing evidence on the economic prowess of young firms (see, e.g., Haltiwanger et al., 2016) and newly posits them as key propagators of R&D policies.

While small firms also feature a high relative Bang for the Buck, this result is entirely driven by the fact that a sizeable fraction of small businesses is young. In fact, there is essentially no advantage of focusing policy on small-old businesses as they deliver the same Bang for the Buck as subsidizing all firms indiscriminately.

[3]Heterogeneity matters also for the Buck. While responsive firms also tend to be those with larger R&D shares, this relationship is quantitatively weaker.

In the last step of our analysis, we show that accounting for knowledge spillovers, dynamics, or borrowing constraints does not change our results. To incorporate knowledge spillovers, we extend our analytical framework to allow firm growth to be partly driven by R&D investment of other businesses. Empirically, we build on Bloom et al. (2013) and measure the technological proximity of firms using citation-weighted patents and their technological classification. While accounting for spillovers favors R&D-intensive businesses, the relative ranking of firm groups does not change. Similarly, our results are robust to accounting for dynamics. Specifically, classifying businesses into previously discussed groups and following them over time shows that young and fast-growing firms remain to deliver the strongest Bang for the Buck even a decade into the future. Finally, in Appendix B.3 we extend our framework with borrowing constraints and show that while more generous innovation subsidies also alleviate credit constraints, this effect is dwarfed by firms' primary interest in taking advantage of cheaper R&D costs.[4]

Related Literature. Our paper is related to several strands of the literature. First, it connects to empirical studies of how firms and innovators respond to (R&D) tax changes (see, e.g., Hall, 1993; Bloom et al., 2002; Moretti et al., 2019; Akcigit et al., 2022a; Fieldhouse and Mertens, 2023; Appelt et al., 2025). We contribute to this literature by developing a novel empirical framework allowing us to measure firms' responsiveness (micro-elasticities) using sufficient statistics—R&D-to-profit ratios—which are readily observable in balance sheet data. In addition, we extend this literature by showing that firms' responsiveness estimated using established DiD methods is indeed closely tied to our sufficient statistics.

Second, we build on a long tradition of endogenous growth models (see, e.g., Grossman and Helpman, 1991; Aghion and Howitt, 1992; Klette and Kortum, 2004; Lentz and Mortensen, 2008; Luttmer, 2010; Akcigit and Kerr, 2018; De Ridder, 2024). Relative to these papers, we use the trade-offs inherent to firms' optimal R&D decisions which lie at the heart of such growth models to derive our sufficient statistics and then validate them with established DiD estimates from micro-data and with model simulations.[5]

Next, our paper relates to recent model-based evaluations of R&D policies (see e.g. Acemoglu et al., 2018; Atkeson and Burstein, 2019; Akcigit et al., 2022b). In contrast to these studies, our sufficient statistics approach lends itself to direct aggregation of observed firm-level heterogeneity without the need for simplifying parameterizations. We then use our framework to show how differences across firms observed in the data, includ-

[4]Note also that Ottonello and Winberry (2025) estimate that the majority of innovation in the U.S. is performed by unconstrained firms.

[5]Similar to Akcigit et al. (2022b), we consider only intensive margin decisions of firms. Therefore, our framework can be viewed as an approximation around the prevailing distribution of firms and associated balanced growth path, similar to, e.g., Atkeson and Burstein (2019). Conceptually, our approach is close to Chiavari et al. (2025) who provide an analytical decomposition of the aggregate return on capital into the contributions of firm-level outcomes.

ing in their responsiveness to policies, shape the relative effectiveness of R&D incentives.

Finally, our paper also connects to the literature studying how firm heterogeneity affects the macroeconomy (see e.g. Hopenhayn and Rogerson, 1993; Restuccia and Rogerson, 2008; Haltiwanger et al., 2016; Carvalho and Grassi, 2019; Sterk ⓡ al., 2021). We contribute to this literature by focusing on aggregate growth and highlighting the prominent role of young, fast-growing firms in propagating R&D policies.

Paper Organization. Section 2 presents our framework and provides key analytical results. Sections 3 and 4 describe the data and validate our assumptions and predictions. Finally, Section 5 presents our main empirical application and the final section concludes.

2 Analytical Framework

The central goal of this paper is to understand—both theoretically and empirically—how R&D subsidies targeted at *individual firms* impact *aggregate growth*. This section develops an analytical framework underlying our analysis. All proofs are deferred to Appendix A.

2.1 Environment

To ease the notation, we use lower-case letters to denote firm-level variables and upper-case letters to denote aggregates.

Production. We consider an environment in which heterogeneous firms i combine to produce aggregate nominal output, Y:

$$Y = \sum_i y_i = \sum_i p_i q_i, \tag{1}$$

where p_i and q_i are the respective firm-level price and quantities.

Aggregate and Firm-Level Growth. In this setting, real aggregate growth, G, is given by:

$$G = \sum_i m_i g_i, \tag{2}$$

where $m_i = y_i/Y$ is the market (sales) share of firm i and where $g_i = \mathrm{d}q_i/q_i$ is firm-level real output growth. Therefore, aggregate growth G is a weighted average of firm-level growth rates, where the weights are firms' sales shares (Domar weights).

We assume that—in order to grow—firms invest into research and development (R&D). In particular, firms optimally choose innovation rates, x_i, which come at a (potentially firm-specific) cost $\widetilde{s}_i(x_i)$. While firm growth is proportional to x_i, i.e. $\frac{\partial g_i}{\partial x_i}\frac{x_i}{g_i} = 1$, R&D costs are convex in the innovation rate, i.e. $\frac{\partial \widetilde{s}_i(x_i)}{\partial x_i}\frac{x_i}{\widetilde{s}_i(x_i)} = \psi_i > 1$.

Government R&D Incentives. We assume that the government subsidizes a fraction, τ, of firms' R&D costs.[6] In this setting, firms' total expenditures on R&D are given by $s_i = (1-\tau)\widetilde{s}_i(x_i)$. Let us denote the aggregate expenditures on R&D invested by firms as:

$$S = \sum_i s_i. \tag{3}$$

Aggregate expenditures on R&D funded by the government are then given by:

$$T = \sum_i \frac{\tau}{1-\tau} s_i. \tag{4}$$

Micro-Elasticities. A key object of our analysis will be the elasticity with which firms' growth rates respond on impact to changes in government innovation subsidies.[7] We refer to this object as the "micro-elasticity:"

$$\epsilon_i = \frac{\partial x_i}{\partial \tau}\frac{\tau}{x_i} = \frac{\partial g_i}{\partial \tau}\frac{\tau}{g_i}. \tag{5}$$

2.2 Bang for the Buck

With the above structure at hand, we are now ready to define our main statistics.

Definition **1** (Bang for the Buck)**.** *Consider a permanent change to the subsidy rate,* $d\log(\tau)$*. Then,*
(i) The Bang, B, is the associated impact response of the aggregate growth rate:

$$B = \frac{\mathrm{d}G}{\mathrm{d}\log\tau}.$$

(ii) The Buck, C, is the associated impact response of aggregate government spending on R&D support:

$$C = \frac{\mathrm{d}\log T}{\mathrm{d}\log\tau}.$$

(iii) The Bang for the Buck, A, is the ratio of the Bang and the Buck:

$$A = \frac{B}{C} = \frac{\mathrm{d}G/\mathrm{d}\log\tau}{\mathrm{d}\log T/\mathrm{d}\log\tau}.$$

[6] We assume that the government funds R&D subsidies by levying lump-sum taxes on the household, thereby abstracting from possible distortionary effects of raising revenues.

[7] Let us emphasize here that we derive our framework in partial equilibrium. Therefore, our approach should be viewed as an approximation around the prevailing distribution of firms and associated balanced growth path, similar to e.g. Atkeson and Burstein (2019). Section 5.4 provides a discussion of how general equilibrium and firm selection may impact our results.

Intuitively, following a permanent change in the R&D subsidy rate, the Bang for the Buck measures the extent to which aggregate growth changes (on impact) per dollar spent on the R&D policy change. Next, we turn to decomposing the Bang for the Buck into underlying firm-level components.

Components of the Bang for the Buck. The following proposition describes how the Bang and the Buck relate to firm-level variables. In anticipation of our empirical application, we also define the Bang and the Buck for mutually exclusive groups of firms Ω_k, where the set of all firms is given by $\Omega = \cup_k \Omega_k$. In addition, for tractability we also make the following assumption:

Assumption **1** (Common R&D cost elasticity)**.** *Assume that the convexity of R&D costs is common to all firms, i.e. $\psi_i = \psi$ for all i.*

In Section 4, we directly estimate ψ from the data and show that it indeed varies relatively little in the cross-section.[8]

Proposition **1** (The Bang and the Buck)**.** *In the environment described above, the Bang and the Buck are given by:*

$$B = \sum_k B_k = \sum_{i \in \Omega} m_i g_i \epsilon_i, \tag{6}$$

$$C = \sum_k C_k = \sum_{i \in \Omega} r_i \left(1 + \psi \epsilon_i\right), \tag{7}$$

where $m_i = y_i/Y$ are firm-level market (sales) shares, $r_i = s_i/S$ are firm-level R&D shares, k indicates mutually exclusive firm groups Ω_k and where $B_k = \sum_{i \in \Omega_k} m_i g_i \epsilon_i$ and $C_k = \sum_{i \in \Omega_k} r_i \left(1 + \psi \epsilon_i\right)$ are, respectively, the group-specific Bang and Buck.

The first part of Proposition 1 makes clear that the Bang is determined by a combination of firms' sales shares, m_i, their growth rates, g_i, and their micro-elasticities, ϵ_i. Intuitively, following an R&D poicy change, an individual firm can leave a mark on aggregate growth only if it is sufficiently large, fast-growing and responsive to the policy change.

The second part of Proposition 1 shows that the Buck is a combination of firms' R&D shares, r_i, and micro-elasticities, ϵ_i. Intuitively, a given policy change will be particularly expensive if firms respond strongly to it and if affected businesses already receive a large chunk of R&D incentives. The latter, given by firms' R&D shares can be viewed as a measure of "policy exposure."

[8]Moreover, note that Assumption 1 does not preclude firm-level R&D cost functions to differ across businesses. This can happen if firms differ in the efficiency with which they innovate (level of the cost function).

Put together, Proposition 1 highlights that understanding the *aggregate* impact of changes in R&D incentives does not depend solely on firms' responsiveness (micro-elasticities). Instead, it is necessary to take into account the joint distribution of four firm-level elements: (i) micro-elasticities, ϵ_i, (ii) sizes, m_i, (iii) growth rates, g_i, and (iv) R&D expenditures, r_i. The following proposition makes explicit how these firm-level factors drive the Bang and the Buck.

PROPOSITION **2** (Components of the Bang and the Buck)**.** *(a) The Bang (6) can be expressed as:*

$$B = \sum_k g_k m_k \bar{\epsilon}_k \theta_k^g, \tag{8}$$

where each group-specific Bang is driven by four components:

i) growth (size-weighted): $g_k = \sum_{i \in \Omega_k} \frac{y_i}{Y_k} g_i$

ii) size: $m_k = \sum_{i \in \Omega_k} y_i / Y = Y_k / Y$

iii) average micro-elasticity: $\bar{\epsilon}_k = \frac{1}{N_k} \sum_{i \in \Omega_k} \epsilon_i$

iv) heterogeneity (in growth): $\theta_k^g = 1 + \frac{\text{cov}(g_k^y, \epsilon_k)}{\bar{g}_k^y \bar{\epsilon}_k}$,

where N_k *denotes the number of firms in group* k *and where* $\bar{g}^y = \frac{1}{N_k} \sum_{i \in \Omega_k} y_i g_i$ *are average output changes in group* k*.*

(b) The Buck (7) can be expressed as driven by three components:

$$C = \sum_k r_k \left(1 + \psi \bar{\epsilon}_k \theta_k^s\right), \tag{9}$$

where each group-specific Buck is driven by three components:

i) R&D "exposure": $r_k = \sum_{i \in \Omega_k} s_i / S = S_k / S$

ii) average micro-elasticity: $\epsilon_k = \frac{1}{N_k} \sum_{i \in \Omega_k} \epsilon_i$

iii) heterogeneity (in R&D): $\theta_k^s = 1 + \frac{\text{cov}(s_k, \epsilon_k)}{\bar{s}_k \bar{\epsilon}_k}$,

where $\bar{s}_k = \frac{1}{N_k} \sum_{i \in \Omega_k} s_i$ *are average R&D expenditures in group* k*.*

Proposition 2 puts forward a convenient decomposition of the Bang and the Buck into average values of each of the components and the influence of firm heterogeneity. Intuitively, the Bang is higher if responsive firms (those with high ϵ_i) are also large and/or fast growing businesses, i.e. $\text{cov}(g^y, \epsilon) > 0$ and, therefore, $\theta^g > 1$. On the other hand, the Buck is higher if responsive firms are also R&D intensive, i.e. $\text{cov}(s, \epsilon) > 0$ and, therefore, $\theta^s > 1$. This highlights that the ability to measure micro-elasticities at the firm-level is crucial for understanding the aggregate effect of R&D incentives. We turn to this next.

2.3 Firm-Level Micro-Elasticities

The previous paragraphs put forward the potential importance of accounting for firm-level heterogeneity in responsiveness to R&D subsidy changes. However, existing estimates typically do not offer such a high degree of granularity. Instead, micro-elasticities are typically estimated for all firms as a whole or for broad groups of businesses (e.g. small versus large).

In this subsection, we propose a novel approach for obtaining measures of micro-elasticities for *individual firms*. In contrast to existing empirical approaches which rely on geographical or time variation in R&D policies, our methodology rests on key economic tradeoffs firms face when deciding on R&D investment.

Importantly, while these tradeoffs lie at the heart of modern theories of endogenous growth, our approach will *not* require us to solve such complex models with heterogeneous firms. Instead, under a certain condition—which we show holds in the data—we will derive closed form expressions for micro-elasticities, serving as sufficient statistics measurable in existing firm-level datasets.

Optimal R&D Investment. Consider that firms choose R&D investment (firm-level growth) optimally to maximize their value, v_i. The latter is the discounted present value of all future profits:

$$v_i(q_{i,t}) = \max_{g_{i,t}} \sum_{j=0} \beta_i^j \pi_i(q_{i,t+j}),$$

where $\beta_i \in (0,1)$ is a discount factor, potentially also reflecting firm exit, and where $\pi_i(q_{i,t}) = \pi_i^o(q_{i,t}) - s_{i,t}$ are profits with $\pi_i^o(q_{i,t})$ indicating "operational profits" which are, by construction, independent from R&D subsidies. In this setting, R&D investment (firm-level growth) satisfies the following optimality condition:

$$\psi \frac{s_{i,t}}{x_{i,t}} = \sum_{j=1} \beta_i^j \frac{\epsilon_{i,t+j}^{\pi,q}}{1+g_{i,t}} \pi_{i,t+j}. \tag{10}$$

Equation (10) shows that optimal innovation decisions balance marginal costs and benefits of R&D. The former are governed by firms' R&D cost functions. The latter are driven by the effect innovation has on all future profits—summarized in (10) by the (potentially firm-specific) elasticities $\epsilon_i^{\pi,q}$.

In what follows, we use equation (10) to derive firms' responsiveness to a change in R&D subsidies. First, however, we make the following assumption:

ASSUMPTION **2** (Constant R&D-to-profits at the firm-level). *Assume that at the firm level, the ratio of R&D expenditures to profits is constant, i.e.* $s_{i,t}/\pi_{i,t} = s_i/\pi_i$ *for any* t.

While Assumption 2 posits that R&D-to-profits are constant over firms' life-cycles, they may well differ across businesses. Such heterogeneity is governed by firms' individual

R&D cost functions and innovative capacity. In Section 4, we discuss the validity of this assumption and show that it holds in a large class of endogenous growth models. Importantly, we show that it also holds in the data.[9]

The following proposition describes the nature of micro-elasticities at the firm-level.

PROPOSITION **3** (Firm-level micro-elasticities). *Under Assumptions 1 and 2, firm-level micro-elasticities are given by:*

$$\epsilon_i = \frac{s_i}{\pi_i} \frac{1}{\psi - 1}. \tag{11}$$

Proposition 3 shows that firm-level micro-elasticities are proportional to their R&D-to-profit ratio. Intuitively, as R&D becomes cheaper due to more generous subsidies, firms optimally increase their investment into innovation. On the one hand, doing more R&D comes at higher costs now and in the future. On the other hand, it increases expected growth and, therefore, current and future profits. The precise way how firms balance these trade-offs is informative about their underlying "micro-elasticity." If R&D expenditures to profit ratios are constant at the firm-level (Assumption 2), then Proposition 3 states that firms' micro-elasticities boil down to "just" the ratio of these two sides of the balancing act.

Note further that both R&D expenditures and profits are directly observable in readily available firm-level datasets. This makes our measure of micro-elasticities particularly appealing. Therefore, given an estimate of the R&D cost elasticity which we discuss below, our framework is capable of considering the full extent of firm-level heterogeneity observed in the data.[10]

2.4 Spillovers

Up until now, we have assumed that firm-level growth is affected solely by firms' own R&D investment. In this subsection, we extend our analysis to allow for external effects, i.e. spillovers from R&D investment of other firms.[11]

Impact of External R&D. To analyze how spillovers may affect the Bang for the Buck, let us consider that firm-level growth is a combination of growth driven by firms' "own" R&D efforts and growth driven by "external" spillovers from R&D of other firms.

[9] In Appendix E, we generalize our result to cases when R&D-to-profits are not constant at the firm-level (see e.g. Akcigit and Kerr, 2018; Ignaszak and Sedláček, 2025, for examples of such models) and we show that our results do not fundamentally change.

[10] In Appendix B.2 we show that Proposition 3 holds also in models in which R&D investment is risky and when at the firm level R&D-to-profits follow a random walk, rather than being fixed. In addition, Appendix B.1 also provides a brief description of a workhorse model of endogenous growth in which Proposition 3 holds.

[11] At the end of Section 5 and in Appendix B.3, we also consider an extension of our framework which incorporates possible credit constraints on firms.

Formally, we can then write:

$$g_i = \eta_{own} x_i + \eta_{ext} \underbrace{\sum_{j \neq i} \alpha_{i,j} x_j}_{S_i^{ext}}, , \tag{12}$$

where η_{own} and η_{ext} represent the extent to which "own" and "external" innovation rates affect the growth rate of firm i and where $\alpha_{i,j}$ are "technology-proximity and quality-adjustment" weights. The latter summarize not only how closely firm i is related to firm j in terms of their technological fields, but also the quality of innovation in firm j. The following proposition describes the micro-elasticity and the Bang for the Buck under spillovers.

PROPOSITION 4 (Spillovers). *In the environment described above, micro-elasticities with spillovers can be expressed as*

$$\epsilon_i^{spill} = \omega_i \epsilon_i + (1 - \omega_i) \sum_{j \neq i} \sigma_{i,j} \epsilon_j, \tag{13}$$

where $\omega_i = \eta_{own} x_i / g_i$ is the fraction of "own-growth", ϵ_i is defined in (11) and $\sigma_{i,j} = \alpha_{i,j} x_j / S_i^{ext}$ are technology-proximity and quality-adjusted R&D shares of other firms, where $S_i^{ext} = \sum_{j \neq i} \alpha_{i,j} x_j$.
The Bang for the Buck is then given by

$$A^{spill} = \frac{\sum_i m_i g_i \epsilon_i^{spill}}{\sum_i r_i (1 + \psi \epsilon_i)}. \tag{14}$$

Proposition 4 first shows that the micro-elasticity of firm i which takes spillovers into account is a weighted average of the firm's individual micro-elasticity, ϵ_i, and that of all other businesses. However, micro-elasticities of all other businesses influence firm i only to the extent that they are technologically close, $\sigma_{i,j}$, and to the extent that external R&D drives some of firm i's growth, $1 - \omega_i$. Finally, note that the spillover elasticity only enters the Bang, but not the Buck. The reason is that while R&D investment of a given firm may influence any other business, the government always subsidizes each firm only once.

Spillovers and Subsidizing Only a Subset of Firms. Next, let us consider the case when only firms of a particular group, Ω_k, are subsidized. The following proposition shows how we can decompose the group-specific Bang for the Buck into contributions of "own" R&D investment, internal and external spillovers and how all these relate to the case which ignores spillovers.

Proposition 5 (Bang for the Buck with & without Spillovers). *For firm group Ω_k, we can write the Bang for the Buck which accounts for spillovers as*

$$A_k^{spill} = A_k \left(\frac{B_k^{own}}{B_k} + \frac{B_k^{int} + B_k^{ext}}{B_k} \right), \tag{15}$$

where $A_k = B_k / C_k$ is the group-specific Bang for the Buck without spillovers with B_k and C_k defined in (8) and (9), respectively, and where $B_k^{own} = \sum_{i \in \Omega_k} m_i g_i \omega_i \epsilon_i$, is the "own" Bang, $B_k^{int} = \sum_{i \in \Omega_k} m_i g_i (1-\omega_i) \sum_{j \in \Omega_k | \neq i} \sigma_{i,j} \epsilon_j$ is the "internal" Bang and $B_k^{ext} = \sum_{j \in \Omega_{\neq k}} m_j g_j (1-\omega_j) \sum_{i \in \Omega_k} \sigma_{j,i} \epsilon_i$ is the "external" Bang.

The proposition makes clear how accounting for spillovers relates to the Bang for the Buck which ignores the effects firm innovation has on R&D of other businesses. Specifically, the "own" Bang is a scaled-down version of the Bang without spillovers, where the scaling depends on the extent to which firms' drive their own growth, ω_i. The internal Bang is a spillover effect from other firms within the subsidized group k. Finally, the external Bang quantifies how subsidizing firms in group k impacts growth of other firms outside the subsidized group due to technological spillovers.

3 Data and Measurement

A major advantage of our approach is that all objects of interest are measurable with readily available data. Therefore, for our application we choose Compustat, a widely used firm-level dataset which covers publicly traded firms in the U.S. economy. In what follows, we describe the nature of the firm-level information, sample selection and the definitions of firm groups in Compustat. We defer further details to Appendix C.1. The next section validates key assumptions and predictions of our framework and Section 5 applies it to the Compustat data described here.

3.1 Firm-Level Sales, R&D and Profits

As explained in the previous section, our main analysis relies "only" on three firm-level variables: R&D expenditures, sales (growth) and profits. In the Appendix, we show that our main results are unchanged when considering employment growth, instead of sales growth (Appendix F.4), and when employing alternative definitions of firm profits (Appendix F.5).

Sample selection. Our primary sample period is 1980-2019 when R&D coverage is high. During this period, Compustat firms accounted, on average, for 75% of annual aggregate R&D investment and 60% of real GDP.

In our application, we focus on the intensive margin of R&D and, therefore, restrict our attention to firms with non-negative R&D expenditures and profits.[12] In addition, to deal with outliers, we winsorize firms at the top 1% distribution of the R&D-to-profits (s_i/π_i), sales (y_i) and the sales growth rate (g_i). The percentiles are computed as averages in a given industry-period cell to avoid an imbalanced sample that leans on certain industries or periods of time.

Following the convention in the literature, we drop firms in agriculture, finance, insurance, real estate and public utilities. In addition, we drop observations with missing industry classifications, negative sales and observations in which acquisition expenses exceed 10% of revenues. We do the latter in order to control for the mergers and acquisitions for which measured growth may only be the result of acquiring a different business unit.

Measurement. The financial economics literature has long recognized that annual growth rates are a noisy measure of companies' fundamentals (see, e.g., Campbell and Shiller, 1988). We follow the common approach in the literature and average each firm-level variable of interest over a certain time window for all our results. This approach both smooths variation and allows for potential lags between R&D investment and growth. In our baseline specification, we average each outcome over non-overlapping 5-year windows so that the subscript "i" in the theory developed in Section 2 corresponds to a firm-window cell. After this time averaging, we pool all observations.[13]

Firm-level sales shares are given by $m_i = y_i/(\sum_i y_i)$, where we use Compustat `sales` to measure y_i. Similarly, firm-level R&D shares are measured analogously as $r_i = s_i/(\sum_i s_i)$, where s_i is given by Compustat `xrd`.[14] Firm-level growth rates are computed as a simple average of annual growth rates within a given window. Finally, for our main analysis, we define profits as `sales - cogs - xrd`, where `cogs` are costs of goods sold, a measure of variable cost.[15]

[12]Indeed, Dechezlepretre et al. (2023) find that the extensive margin does not respond to R&D incentives and, in our theoretical framework, firms only conduct R&D if they expect positive benefits—proxied by firm profits. Nevertheless, in Appendix F.9, we show that our results remain unchanged even when including firms with negative profits.

[13]For example, s_1 would correspond to arithmetic mean of R&D expenditure of the first firm in our sample in the period 1980 to 1984. The mean R&D expenditures for this firm in period 1985 - 1989 are treated as an independent observation in the pooled dataset. In Appendix F.7 we show that our main results are qualitatively and quantitatively robust to using directly annual data.

[14]In Appendix F.6, we show that our results are unchanged when we, instead, consider nominal GDP and aggregate expenditures on R&D from the National Accounts as measures of aggregate output and R&D expenditures.

[15]Following common practice in the literature (see e.g. De Loecker et al., 2020), we deflate nominal variables with the GDP deflator (2012-based, BEA code A191RD). Note further that since our focus is on governmental policies aimed at R&D, it is precisely accounting-based R&D spending which is of primary interest (in contrast to "undeclared" innovation or innovation output such as patents).

3.2 Micro-Elasticities

As shown in Proposition 3, micro-elasticities are given by $\epsilon_i = s_i/\pi_i \frac{1}{\psi-1}$. As mentioned earlier, in (16) we use `xrd` to measure s_i and `sales - cogs - xrd` to measure π_i. In addition, for our baseline results, we follow the literature and set $\psi = 2$ (see e.g. Hall et al., 2001; Blundell et al., 2002; Bloom et al., 2002). In Section 4 we estimate ψ directly using data on patents and show that it is not statistically different from 2 and that it varies relatively little across industries.

Firms with negative growth rates. Notice that our micro-elasticities are positive, indicating that firms grow faster as a result of cheaper (more subsidized) R&D. However, in the data, some firms report negative growth rates. In order to retain the property that more favorable R&D incentives lead to improved growth (even if initially negative), we define micro-elasticities in the Bang, $B = \sum_i m_i g_i \epsilon_i^B$ as follows:

$$\epsilon_i^B = \begin{cases} \epsilon_i & g_i \geq 0, \\ -\epsilon_i & \text{when } g_i < 0. \end{cases} \tag{16}$$

In this way, an increase in the subsidy rate boosts growth in businesses which are expanding and slows the contractions in firms with negative growth rates.[16] Importantly, note that the Buck remains to feature $\epsilon_i = s_i/\pi_i$, as more favorable R&D incentives increase firms' expenditure on R&D, irrespective of whether they are growing or shrinking.

3.3 Spillovers

Seminal contributions of Jaffe (1986) and Bloom et al. (2013) document sizable technological spillovers between firms. We follow Bloom et al. (2013) and use data on patenting activity to measure technological spillovers between firms.

Technological Proximity. In particular, we use patent data collected by Kogan et al. (2017) which includes the technology classifications, so-called CPC codes.[17] Following Bloom et al. (2013), we use CPC codes to measure closeness between firms in the technology space and to quantify R&D spillovers.

Towards this end, we classify each patent using 3-digit CPC codes into one of 130 technology classes. Next, for each firm we compute the share of patents in all technology

[16] In Appendix F.8, we show that very similar results are obtained when dropping firms with negative growth rates altogether, in which case $\epsilon_i^B = \epsilon_i$.

[17] CPC stands for the Cooperative Patent Classification which is patent classification standard jointly managed by the European Patent Office and the US Patent and Trademark Office. Examples of 3-digit CPC codes include "hydraulic engineering; foundations; soil shifting" (CPC code E02) or "electronic circuitry" (CPC code H03).

classes and compute the un-centered correlation between firm i's and firm j's patent shares denoted by $\widetilde{\alpha}_{i,j}$.[18]

Quality Adjustment. Building on Griffith et al. (2011) and Bloom et al. (2013), we account for firm's importance in the patenting network by weighting each observation by the total patent citations accrued to the given firm.[19]

Towards this end, let c_j denote all citations attributed to patents assigned to a given firm-period cell j. Then, our measure of spillovers benefiting firm i is given by the proximity-weighted citation-adjusted R&D expenses of all other firms $\sigma_i = \sum_j \alpha_{i,j} \frac{s(x_j)^{\frac{1}{\psi}}}{S_i^{ext}}$, where $\alpha_{i,j} = \widetilde{\alpha}_{i,j} \frac{c_j}{C}$ with $C = \sum_j c_j$ marking the total citation count.[20] Intuitively, a firm i benefits from R&D expenses of a firm j if the two firms are patenting in the same set of CPC sectors and if the firm j tends to generate patents with a high citation count.[21]

3.4 Firm Groups

To highlight the heterogeneity present in the data, we consider four groups of firms, commonly discussed in the literature: (i) small and medium-sized enterprises (SMEs), (ii) R&D-intensive firms, (iii) young firms, and (iv) gazelles.

The first two groups are defined by the respective medians of firm size (sales), and R&D-to-sales. In each case, the medians are computed individually in each time period-window in the averaged data or year in the annual data. Young firms are defined as those weakly less than 6 years after their IPO. To define high-growth firms ("gazelles"), we follow Haltiwanger et al. (2016). In particular, gazelles are businesses with an annualized growth rate within our averaging window which weakly exceeds 20%.

4 Validation

In this section, we discuss the empirical relevance of Assumptions 1 and 2. In addition, we make use of existing studies as well as a novel empirical analysis using micro-data to validate the predictions of our analytical framework regarding micro-elasticities.

[18]Formally, $\widetilde{\alpha}_{i,j} = \frac{T_i T_j'}{\sqrt{T_i T_i'}\sqrt{T_j T_j'}}$, where T_i is 130 element vector in which each element corresponds to the number of patents granted to firm i in a given CPC class. Further details on the patent data, its matching to Compustat and the construction of technology-proximity weights can be found in Appendix C.1.

[19]Griffith et al. (2011) use patent citations as a direct measure of technological spillovers between inventors.

[20]Note that firms' innovation rates are given by $x_j = s_j^{-1}(s_j(x_j))$, where the firm-specific function $s^{-1}(y) \propto y^{1/\psi}$ is the inverse of a firm's R&D cost function which is proportional to the firm's R&D expenditure to the power of $1/\psi$.

[21]In Appendix F.10, we document that adjusting spillovers for patent value estimated in Kogan et al. (2017), instead of citation counts, leads to very similar results.

4.1 R&D Cost Elasticity

Under Assumption 1, our framework considers common R&D cost elasticities across firms. For our baseline results in Section 5, we follow the existing literature and consider $\psi = 2$ (see e.g. Hall et al., 2001; Blundell et al., 2002; Bloom et al., 2002). In what follows, we use Compustat to directly estimate the R&D cost elasticity on average and across 2-digit industries.

R&D cost elasticity, ψ: Average for the U.S. economy. Following Hall and Ziedonis (2001), we use information on firm-level R&D expenditures (s_i) and innovation rates (x_i)—proxied by patent applications—to estimate the R&D cost elasticity from the following regression:[22]

$$\Delta \log \left(\text{patents}_{i,t}\right) = \delta_{i,t} + \beta \Delta \log \left(\text{R\&D}_{i,t}\right) + \eta_{i,t}, \tag{17}$$

where $\text{patents}_{i,t}$ is the total number of patents for which a firm i applied in period t, $\delta_{i,t}$ mark fixed effects (sector, time, firm or their combinations, depending on the specification), $\text{R\&D}_{i,t}$ represents the sum of a firm's R&D expenses in period t and $\eta_{i,t}$ is a residual term. As in our main analysis, we aggregate inputs and outputs over non-overlapping 5-year windows in order to account for time-to-build in innovation.

The estimated coefficient β in regression (17) is then a measure of the R&D cost elasticity. In particular, our assumptions about firms' R&D cost functions, namely that $\frac{\partial s_i}{\partial x_i} \frac{x_i}{s_i} = \psi$, imply that $\beta = 1/\psi$. Table 1 shows the results. For all specifications presented in the table, we cannot reject the null hypothesis that $\psi = 2$—consistent with the existing literature.

R&D cost elasticity, ψ: Heterogeneity across industries. To gauge the extent of potential heterogeneity in R&D cost elasticities across industries, in Appendix D.1 we estimate regression (17) individually in each 2 digit SIC sector. The results show that for the vast majority (77%) of the SIC industries we cannot reject the null hypothesis that $\psi = 2$. Therefore, in our application to the data in Section 5 we will consider a common cost elasticity of $\psi = 2$.

4.2 R&D-to-Profits

Assumption 2 considers cases in which R&D to profit ratios are constant at the firm level, though possibly heterogeneous across firms. We begin by inspecting R&D to profit ratios in our Compustat data. Next, we also discuss how this assumption relates to a wide range of existing endogenous growth models.

[22] Note that we count only patents that were eventually granted.

Table 1: R&D cost elasticity in the data

	(I)	(II)	(III)
Δ log R&D expenses	0.54	0.53	0.46
	(0.03)	(0.03)	(0.04)
sector-time fixed effects	✓		
sector fixed effects		✓	
time fixed effects		✓	✓
firm fixed effects			✓
Observations	3,217	3,217	3,217
R^2	0.40	0.37	0.73
Within R^2	0.22	0.22	0.16

Note: Estimated coefficient β in regression (17). The coefficient corresponds to the elasticity of patents (innovation output) to the R&D expenses (innovation inputs). Sample consists of US public firms matched to patent data in Kogan et al. (2017). We restrict the sample to the period 1970 - 2019. To allow for time-to-build in innovation, we aggregate data into non-overlapping 5 year windows. Each observation corresponds to one firm-window cell. If a firm exists for less than 5 years, we retain the firm in the sample and use all years in which the firm is observed.

Assumption 2 in the data. To verify the assumption empirically, we utilize the baseline Compustat sample and estimate the following regression:

$$\log\left(\frac{s_{i,t}}{\pi_{i,t}}\right) = \delta_{i,t} + \alpha t + \beta t^2, \tag{18}$$

where $\delta_{i,t}$ marks fixed effects (sector, firm, cohort, and their combinations depending on specification) and t denotes time. When we condition on firm or cohort fixed effects, the regression captures the average evolution of the R&D-to-profits ratio over firms' life-cycles.

Table 2 shows the results which indicate that, at the firm-level, R&D-to-profits do not change noticeably over time. While some of the coefficients are statistically significant, none of them are quantitatively large. This can be seen from both the extremely low magnitudes of the within R^2 statistic and the low point estimates.

Note further that the estimated variation over firms' life-cycles is dwarfed by differences in R&D-to-profit ratios in the cross-section of firms. In particular, the (statistically insignificant) point estimate in our preferred specification with firm fixed effects (column I in Table 2) would suggest that R&D to profit ratios change by about 5% every five years. Given an average R&D to profit ratio of about 20%, this amounts to an increase of 2 percentage points over a decade. By comparison, the inter-quartile range of R&D-to-profit ratios in the cross-section of firms in our sample lies between 4% and 30%.

Therefore, the dispersion in the R&D-to-profit ratios is primarily driven by cross-sectional differences, rather than changes occurring over firms' life-cycles. This is precisely

Table 2: Within-firm variation in R&D-to-profits ratio

	(I)	(II)	(III)
time	0.050	0.015	0.069
	(0.031)	(0.013)	(0.008)
time^2	-0.001	0.0004	-0.002
	(0.002)	(0.0007)	(0.0005)
firm fixed effects	✓		
cohort fixed effects		✓	
sector fixed effects		✓	✓
Observations	12,234	12,234	12,234
R^2	0.90	0.34	0.31
Within R^2	0.007	0.001	0.01

Note: The dependent variable is $\log\left(\frac{s_{j,t}}{\pi_{j,t}}\right)$ which implies that we only consider firms with positive R&D expenses and positive profits. Both outcomes are averaged over non-overlapping 5-year windows and "time" corresponds to the window index.

the type of heterogeneity that our framework is designed to account for.[23]

Assumption 2 in existing models. Assumption 2 holds not only in Compustat data—as shown in the previous paragraphs—but also in a wide range of existing growth models (see e.g. Klette and Kortum, 2004; Luttmer, 2007; Lentz and Mortensen, 2008; Luttmer, 2010; Mukoyama and Osotimehin, 2019).

The key reason behind this property is the assumption of "perfect scaling." In such models, R&D costs scale one-for-one with firm size, resulting in constant shares of costs in profits (see Akcigit and Kerr, 2018, for a discussion and an example of a framework which deviates from perfect scaling).[24] Put together, Assumption 2 holds not only in the data, but also sits firmly within a very broad range of existing endogenous growth models.

4.3 Estimates of Micro-Elasticities in Existing Studies

A key advantage of our methodology is the ability to estimate micro-elasticities at the firm-level. In what follows, we document that our methodology does in fact deliver estimates which are very close to those based on existing empirical approaches which rely on (geographical or time) variation in R&D policies.

[23]In Appendix D.1 we provide further robustness checks, including estimates using annual data, rather than 5-year averages. These robustness checks indicate that the averaged data provide an upper bound on the magnitude of the time trends in the s/π ratio.

[24]In Appendix E, we consider the model in Ignaszak and Sedláček (2025) which deviates from perfect scaling and show numerically that micro-elasticities predicted by our framework remain to proxy firms' true responsiveness to R&D incentives well.

Table 3: Tax elasticity of R&D expenditures: Existing studies and our approach

Estimate/Firm group	All	Small	Medium	Large
Appelt et al. (2025), ϵ_s	0.43—0.60	0.94—1.29	0.78—1.03	0.19—0.31
Proposition 3, $\epsilon_s = \psi\bar{\epsilon}$	0.50	1.22	0.78	0.28

Notes: The table presents estimates of micro-elasticities. The top row reports the range of estimates of the elasticity of firms' R&D expenditures with respect to their price (tax) from Appelt et al. (2025), Tables 6 and 9, respectively. The latter estimates are based on Orbis data from 11 countries and cross-country variation R&D subsidies. The bottom row uses the same Orbis country sample and the approach described in Proposition 3. "All" refers to all businesses in the sample, while "Small", "Medium" and "Large" are given by firms with 1-49, 50-249 and 250+ employees, following the definitions in Appelt et al. (2025). We assume that $\psi = 2$.

Methodology. We follow a recent study by Appelt et al. (2025) in which the authors estimate the elasticity of R&D expenditures with respect to their (tax) price, i.e. $\epsilon_s = \frac{\partial \tilde{s}}{\partial \tau}\frac{\tau}{\tilde{s}}$. They do so using firm-level data and cross-country variation in R&D incentives. The sample consists of 11 OECD countries which offered some form of R&D tax incentive in the 2000-2021 sample period.[25] The baseline estimates of ϵ_s range between 0.43 and 0.6. Very similar values have also been found in a range of other existing studies (see e.g. Hall and Reenen, 2000; Bloom et al., 2002; Thomson, 2017; Appelt et al., 2019).

In our framework, the elasticity of R&D expenditures to changes in subsidy rates is given by $\epsilon_s = \bar{\epsilon}\psi$, where $\bar{\epsilon}$ is the average micro-elasticity of the given set of firms in the sample. Therefore, to compare the estimates in Appelt et al. (2025), we use firm-level data from Orbis for the same set of 11 countries. Aside from computing the average micro-elasticity for all firms, we also use the definitions in Appelt et al. (2025) to investigate how micro-elasticities differ across broad firm size groups (small, medium and large businesses).

Results. Table 3 summarizes the results. While the first row reports the range of estimates of ϵ_s from Appelt et al. (2025), the bottom row computes the same elasticities using Proposition 3 (and assuming $\psi = 2$). The first column reports values for all firms on average and the remaining columns show the same for small, medium and large businesses as defined by Appelt et al. (2025).

4.4 Novel Estimates of Micro-Elasticities

In what follows, we use Australian administrative micro-data to provide novel evidence on firms' micro-elasticities. In particular, we make use of a difference-in-differences method-

[25]The country sample includes Australia, Belgium, Czechia, France, Italy, the Netherlands, New Zealand, Norway, Portugal, Slovakia and Sweden. The baseline estimates do not account for R&D incentive uptake since we do not have that information in our micro-data. See Appendix C.3 for details pertaining to the Orbis dataset.

ology around a major R&D policy change in Australia in 2012 to estimate firms' average micro-elasticity. Importantly, we also leverage the administrative data to *directly* estimate that the estimated micro-elasticity varies one-for-one with firms' R&D-to-profit ratios, in accordance to our theory. We defer further details on the institutional background, data and methodology to Appendix D.3.

Institutional Background. Like many other countries, Australia provides R&D support to eligible businesses. Prior to 2012, businesses with sales exceeding AUD5 million could deduct 125% of R&D expenditures from their taxable income. Throughout the sample period, the corporate tax rate was 30% implying an effective R&D subsidy of $1.25 \times 0.3 = 0.375$.

In 2012, the government implemented a reform through which the size threshold increased and the R&D deduction was replaced by a tax offset. In particular, firms with sales below AUD20 million became eligible for a 45% tax offset. By contrast, businesses with sales exceeding AUD20 million were eligible for a 40% tax offset.

Evidence from a policy change in Australia: Methodology. In the setting described above, firms with sales between AUD5 million and AUD20 million experienced a 20% increase in the effective R&D subsidy rate (changing from 0.375 to 0.45). We will, therefore, compare R&D expenditure among these businesses to firms with sales exceeding AUD20 million. For this latter group, the effective R&D tax subsidy increased by only 6.7% (from 0.375 to 0.4).

Average Micro-Elasticity. In particular, focusing only on firms with sales exceeding AUD5 million in years 2011 and 2012, we estimate the following regression:

$$\log(R\&D)_{i,t} = \alpha + \beta_1 \mathbb{1}_{\$5-20M} \times \mathbb{1}_{2012} + \gamma_1 \mathbb{1}_{\$5-20M} + \delta_1 \mathbb{1}_{2012} + \lambda X_{i,t} + u_{i,t}, \tag{19}$$

where $R\&D_{i,t}$ are expenditures on research and development at firm i in year t, $\mathbb{1}_{\$5-20M}$ is an indicator function equal to 1 for firms with sales between AUD5 million and AUD20 million, $\mathbb{1}_{2012}$ is an indicator function equal to 1 for the post-reform year 2012, $X_{i,t}$ are other controls and $u_{i,t}$ are residuals.

The coefficient of interest is β_1 which measures the average percent change (between 2012 and 2011) in R&D expenditures of firms with AUD5 $-$ 20 million sales, relative to businesses with sales exceeding AUD20 million.[26] As such, this coefficient (after appropriate transformations described below) provides an estimate of firms' micro-elasticities.

[26] The identifying assumption is that in the absence of the policy reform, R&D expenditures of firms with AUD5 $-$ 20 million in sales and those with AUD20+ million in sales would have changed by the same amount (parallel trends). As we show in Appendix D.4, a placebo treatment estimating (19) prior to the policy change delivers a β_1 coefficient which is statistically not different from zero. This provides support that the parallel trends assumption holds in our data.

Heterogeneity in Micro-Elasticities. In addition to estimating the average micro-elasticity—as explained above—the Australian data gives us the opportunity to *directly* test our key prediction. Namely, to gauge whether firm-level micro-elasticities are indeed governed by the R&D-to-profit ratio, as our theory predicts, we generalize (19) to consider this interaction.

Towards this end, let us first use e_i to denote the R&D-to-profit ratio of firm i relative to the sample mean, i.e. $e_i = s_i/\pi_i - 1/N \sum_i s_i/\pi_i$. Then, the relationship between the firm-level micro-elasticity and firms' R&D-to-profit ratios can be estimated by interacting our treatment indicators with e_i: $(\beta_1 + \beta_2 e_i)\, \mathbb{1}_{\$5-20mil} \times \mathbb{1}_{2012}$. In this case, β_1 is the relevant estimate for the average firm (for which $e = 0$) and β_2 estimates how the average micro-elasticity changes with firms' R&D-to-profits. Formally, we estimate the following generalization of regression (19):

$$\begin{aligned}\log(R\&D)_{i,t} =& \alpha + (\beta_1 + \beta_2 e_i)\, \mathbb{1}_{\$5-20M} \times \mathbb{1}_{2012} \\ &+ (\gamma_1 + \gamma_2 e_i)\, \mathbb{1}_{\$5-20M} + (\delta_1 + \delta_2 e_i)\, \mathbb{1}_{2012} + \eta e_i + \lambda X_{i,t} + u_{i,t}. \qquad (20)\end{aligned}$$

Regression Results: Average Micro-Elasticities. To estimate regressions (19) and (20), we make use of Australia's administrative firm-level data—the Business Longitudinal Analysis Data Environment (BLADE)—developed by the Australian Bureau of Statistics (ABS). Within BLADE, we are able to connect information on firms' R&D expenditures, profits, sales and (3-digit) industries. While R&D represents expenditures subject to the R&D subsidy, profits are operating profits, as reported in the firm's tax statements.

Table 4 presents the regression results. The first two columns show β_1 estimated for different specifications of (19). The results indicate that, on average, businesses with sales between AUD5 and AUD20 million increased their R&D expenditures by about 20 percent more relative to firms with sales over AUD20 million.

To compare this estimate to predictions from our framework, we first convert the estimated β_1 coefficients to average R&D expenditure elasticities by using the information on the relative increases in subsidy rates, $\epsilon_s = \hat{\beta}_1/(\Delta \log \tau)$. Focusing on our preferred specification (column II in Table 4) and using the estimated standard errors to compute an upper and lower bound, our results suggest that the R&D expenditure elasticity lies between $\epsilon_s \in (1.17, 1.88)$ with a point estimate of about 1.53.[27]

Next, to compute the R&D expenditure elasticity using our approach, we employ the fact that $\epsilon_s = \psi\bar{\epsilon}$ and lean on Proposition 3 which states that $\bar{\epsilon} = \frac{1}{N}\sum_i \epsilon_i = \frac{1}{\psi-1}\frac{1}{N}\sum_i \frac{s_i}{\pi_i}$. The average R&D-to-profit ratio among the AUD5-20 million sales firms is 0.692. There-

[27] To convert the point estimate of β_1 to an R&D expenditure elasticity, we divide by 0.133 which represents the percentage difference in policy rate increases between the treatment and control groups (0.45/0.375 relative to 0.4/0.375). We obtain the lower and upper bound as $(0.203 \pm 0.047)/0.133$.

Table 4: Elasticity of R&D expenditures: BLADE

	(I)	(II)	(III)	(IV)
AUD5-20 mil. sales × 2012, β_1	0.202	0.203	0.247	0.250
	(0.048)	(0.047)	(0.050)	(0.049)
AUD5-20 mil. sales × 2012 × e, β_2			0.212	0.224
			(0.075)	(0.076)
Control for sales		✓		✓
Observations	2820	2820	2820	2820

Notes: The table reports the regression results from (19) and (20). Columns refer to different specifications (with and without sales and with and without R&D to profits as a control). The regression sample includes firms in years 2011 and 2012 with sales exceeding AUD5 million, those that report positive R&D expenditure, positive operating profits and remain in the same sales category across both 2011 and 2012. Standard errors in brackets are clustered at the firm level.

fore, using $\psi = 2$, our approach yields an R&D expenditure elasticity of $\epsilon_s = 1.38$ which falls within the above bounds estimated in the data.

Regression Results: Estimated Micro-Elasticities and R&D-to-Profits. The previous paragraphs show that our methodology and established difference-in-differences approaches provide very similar estimates of firms' *average* micro-elasticities. We now turn to analyzing the heterogeneity in micro-elasticities.

Specifically, our methodology predicts that there is a tight relation between firms' micro-elasticities and their R&D-to-profit ratios. In fact, the estimated micro-elasticities should move one-for-one with firms' R&D-to-profits ratios (conditional on ψ). This is because, according to our theory (see Proposition 3), R&D-to-profit ratios are sufficient statistics for firms' micro-elasticities.

In particular, combining estimates from regression (20) and Proposition 3, we can write $\epsilon_{i,s} = \frac{\hat{\beta}_1 + \hat{\beta}_2 e_i}{\Delta \log \tau} = \psi \frac{s_i}{\pi_i}$.[28] Therefore, our framework predicts that $\frac{\hat{\beta}_2}{\psi \Delta \log \tau} = 1$. Using the estimated standard errors to compute upper and lower bounds, the results in the last two columns of Table 4 suggest that the slope with which firms' estimated micro-elasticities move with their R&D-to-profit ratios lies between $\frac{\beta_2}{\psi \Delta \log \tau} \in (0.56, 1.13)$. Formally, we cannot reject the null hypothesis that $\hat{\beta}_2/(\psi \Delta \log \tau) = 1$.

Overall, results in these last two subsections provide validation for our approach. In particular, our methodology does well in capturing the average elasticity of firms' R&D expenditures with respect to their price estimated using established difference-in-differences designs. Moreover, Table 3 shows that our methodology performs well even for the firm size groups considered in Appelt et al. (2025). Finally, using Australian

[28] In regression (20), we use 3-digit industry means in computing e_i. However, the results are similar when using the sample mean across all firms.

Table 5: Components of the Bang and the Buck: Descriptive Statistics

		Percentile				Cumulative Percentile		
	mean	10^{th}	50^{th}	90^{th}		50^{th}	90^{th}	99^{th}
Growth, g	0.09	-0.11	0.04	0.29	Sales share, m	0.98	0.80	0.38
Micro-elasticities, ϵ	0.21	0.02	0.11	0.45	R&D share, r	0.99	0.85	0.47

Notes: The table reports summary statistics for the baseline sample of U.S. public firms used in the analysis of Bang for the Buck in the main text. We restrict attention to firms with positive elasticity ϵ. The sample period is 1980 to 2019. Firm level data is averaged using non-overlapping 5-year windows. Each observation in the dataset corresponds to a firm-window cell.

administrative data, we *directly* estimate that firms' micro-elasticities co-move with their R&D-to-profits ratios one-for-one—validating a key prediction of our framework.

5 Application

This section applies our theoretical framework to readily available firm-level data from the U.S.—Compustat. In what follows, we report results in three stages. First, we quantify the importance of firm heterogeneity in driving the Bang for the Buck. Next, we report the relative Bangs for the Buck for our groups of firms. Finally, we discuss how accounting for spillovers and dynamics affects our results.

5.1 Importance of Firm Heterogeneity

A key advantage of our methodology is the ability to measure micro-elasticities at the *firm-level*. This, in turn, enables us to quantify how the interplay between micro-elasticities and firm size, growth and R&D expenditures influences the *aggregate* Bang for the Buck.

Heterogeneity in Drivers of the Bang and the Buck. Table 5 summarizes the degree of heterogeneity in key driving forces of the Bang and the Buck. Specifically, the first column shows average values for firm-level growth and micro-elasticities. Businesses in our baseline sample grow at an average rate of 9 percent (per 5 years)—noting that this includes businesses which shrink over time.[29] The average micro-elasticity (R&D-to-profit ratio) is 21 percent in our sample where, as the reader will recall, our baseline sample excludes businesses with negative profits.

[29]Note that the Bang uses size-weighted growth rates. The average value of the latter is reported in Table 7.

However, as the next three columns of Table 5 highlight, these averages hide much heterogeneity. In particular, the bottom 36 percent of firms do not grow. In contrast, average growth in the top decile of fastest growing firms is 29 percent. Similarly, there is also large heterogeneity in firms' micro-elasticities with a difference between 90th and 10th percentiles of 43 percentage points.

Finally, using cumulative percentiles, the last three columns show the high levels of sales and R&D concentration observed in the data. Specifically, the top 1 percent firms account for 38 (47) percent of all sales (R&D expenditures) in our baseline sample.

Sources of Cross-Sectional Variation of the Bang and the Buck. The previous paragraphs highlighted the large amount of firm-level heterogeneity in the drivers of the Bang and the Buck. We now turn to quantifying how this heterogeneity influences the cross-sectional dispersion in Bangs and Bucks.

Towards this end, we note that the cross-sectional variation in the (log) Bang and the (log) Buck of individual firms can be written as

$$\text{var}(\ln B_i) = \text{cov}(\ln B_i, \ln m_i) + \text{cov}(\ln B_i, \ln g_i) + \text{cov}(\ln B_i, \ln \epsilon_i) \tag{21}$$

$$\text{var}(\ln C_i) = \text{cov}(\ln C_i, \ln r_i) + \text{cov}(\ln C_i, \ln(1 + \psi\epsilon_i)), \tag{22}$$

Using the above, we can decompose the cross-sectional variation in firm-level Bangs and Bucks into the contributions of heterogeneity in firm (i) size, (ii) growth, (iii) R&D and (iv) micro-elasticities.[30] Table 6 presents the results, where each contribution is expressed in percent of the overall cross-sectional variation of the Bang and the Buck. Two patterns stand out.

First, the results suggest that differences in micro-elasticities account for almost 1/4 of the overall variation in firm-level Bangs. Out of the remaining 3/4, differences in firm size play a dominant role—accounting for over 50 percent of the overall variation.

Second, heterogeneity in micro-elasticities is completely dwarfed by differences in R&D shares when it comes to firm-level Bucks. In particular, cross-sectional variation in R&D expenditure alone accounts for almost 95 percent of the overall differences in firm-level Bucks.

Therefore, while estimating how firms respond to changes in R&D incentives is important in its own right, other factors matter as well when it comes to the impact on *aggregate* outcomes. In particular, firm size and growth enter as major determinants—noting that R&D expenses are closely correlated with firm size (correlation coefficient of 0.75).

[30] In particular, the percentage contributions to the overall cross-sectional variation are simply given by dividing (21) and (22) with the respective left-hand sides.

Table 6: Contributions to Cross-Sectional Variation in the Bang and the Buck (in %)

	Bang	Buck
Sales share, m	51.3	
Growth, g	24.3	
Micro-elasticity, ϵ	24.4	5.2
R&D share, r		94.8

Note: The table shows the relative contribution of each component to the cross-sectional variation in the Bang and Buck (equations (21) and (22)), respectively. Values are expressed in percent of overall cross-sectional variation.

Importance of Firm Heterogeneity for the Aggregate Bang and Buck. We now turn to investigating the importance of firm heterogeneity for the *aggregate* Bang and Buck. As is described in Proposition 2, what matters for the magnitudes of the aggregate Bang and Buck is how micro-elasticities co-vary with firm size, R&D investment and growth rates. The impact of this cross-sectional variation is summarized by the heterogeneity terms θ^g and θ^s, respectively.

We begin by visualizing raw Compustat data to gauge the extent of heterogeneity in the key components of the Bang and the Buck. Figure 1 shows binned scatter plots for all components of the Bang for the Buck. In each panel of the figure, we consider percentiles of the distribution of the variable indicated on the horizontal axis and plot the corresponding sample average of the variables indicated on the vertical axis.

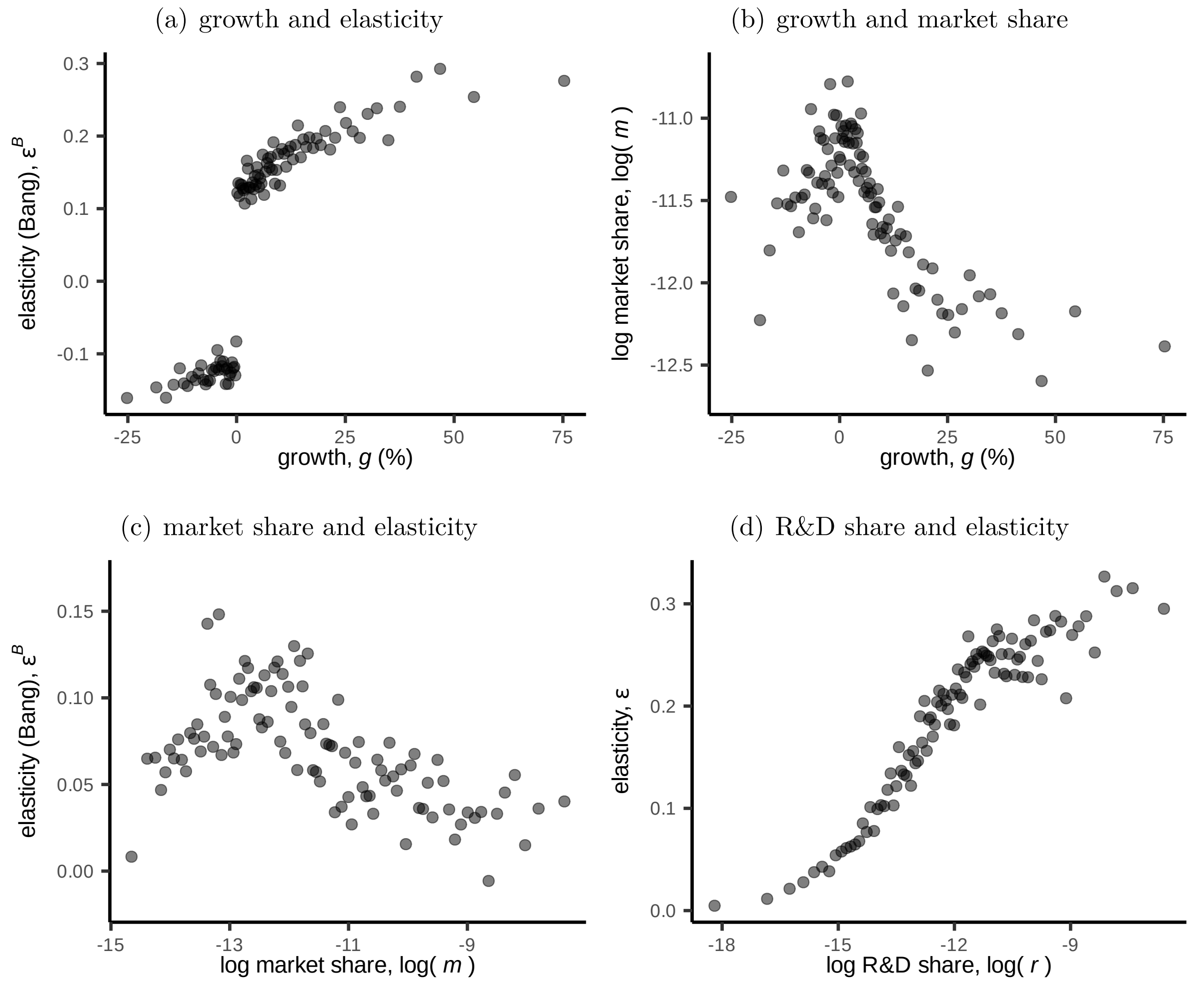
Panel (a) presents the distribution of growth rates, g, and (Bang) micro-elasticities, ϵ^B. As can be seen from the figure, micro-elasticities tend to co-vary positively with growth rates. Next, Panel (b) presents the distribution of growth rates, g, and firm sizes (market shares), m. The plot suggests an inverted-U shape relationship between these variables. In particular, while larger firms are characterized by moderate growth rates, consistent with e.g. Haltiwanger et al. (2013), smaller businesses exhibit either relatively high or low growth. As we discuss in more detail below, this non-linear relationship is partly explained by firm age patterns which will prove important for our further results. Panel (c) shows a similar inverted-U relationship between micro-elasticities, ϵ^B, and firm size (market shares), m, albeit somewhat less pronounced.

Finally, Panel (d) of Figure 1 shows a clear positive relationship between firms' R&D shares and their micro-elasticities, i.e. the two components entering θ^s (see Proposition 2). Quantitatively, $\theta^s = 1.5$ in our baseline sample and, therefore, ignoring firm-level heterogeneity under-estimates the Buck by a factor of about 1.15.[31]

Figure 2 then visualizes how the different components interact to form the Bang and the Buck. In particular, Panel (a) focuses on the Bang and plots micro-elasticities, ϵ^B,

[31]In the case of the Buck, the under-estimation factor is given by $1 + \psi\bar{\epsilon}\theta^s/(1 + \psi\bar{\epsilon})$. With $\psi = 2$, $\bar{\epsilon} = 0.21$ and $\theta^s = 1.5$ this factor is about 1.15.

Figure 1: Distribution of firm-level drivers of the Bang and the Buck

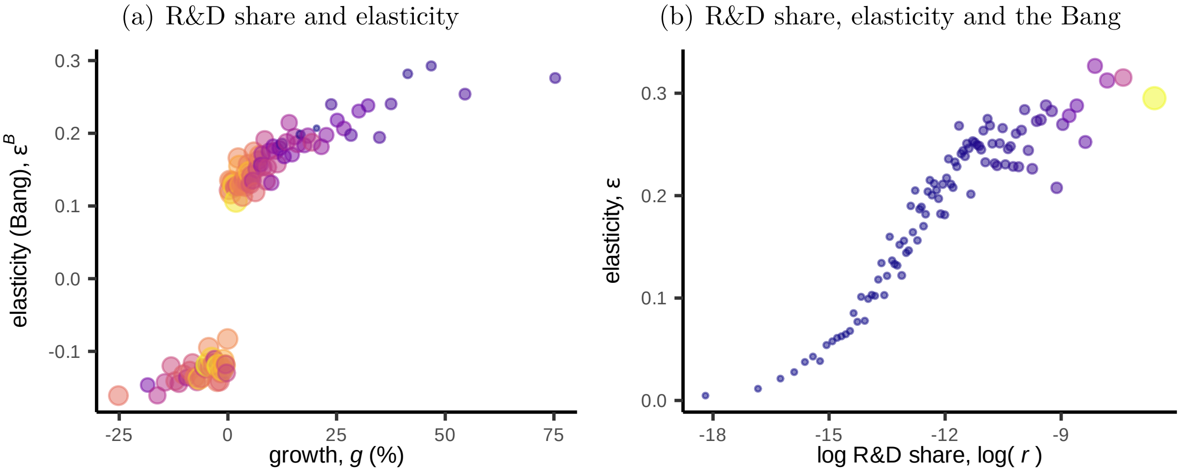


Note: The figure presents binned scatterplots (Cattaneo et al., 2024) for the components of the Bang. Each dot represents 1% of firm-window observations in Compustat data. In each Panel, markers correspond to a percentiles of the variable indicated in the x axis. The corresponding value on the vertical axis is the unconditional sample average among firms within a given percentile.

against growth rates, g, with the size (and color) of each dot indicating the total market share, m. The larger the size (the warmer the color) of each dot, the larger is the market share of firms which find themselves in the given percentile of the growth rate distribution. As can be seen from Panel (a), market shares tend to be concentrated among firms which exhibit a positive connection between micro-elasticities and growth rates. This property of the data will then boost the aggregate Bang—see Proposition 2 and the contribution of heterogeneity, θ^g. Quantitatively, in our baseline sample $\theta^g = 6.7$. Hence, our results suggest that existing approaches which are unable to measure micro-elasticities at the firm-level under-estimate the impact of R&D subsidies on aggregate growth by a factor of almost seven.

Panel (b) of Figure 2 presents the relationship between elasticity ϵ and log R&D share r, but in addition the size (and color) of the dots highlights the magnitude of the

Figure 2: Distribution of firm-level drivers of the Bang for the Buck



Note: The figure presents binned scatterplots (Cattaneo et al., 2024) for the components of the Bang and the Buck. Each dot represents 1% of firm-window observations in Compustat data. In each Panel, markers correspond to a percentiles of the growth rates, g, or the log R&D share, $\log r$. The corresponding value on the vertical axis is the unconditional sample average of the elasticity ϵ among firms within a given percentile. The size and color of the marker in Panel (a) corresponds to firms' market shares, while that in Panel (b) corresponds to the relative Bang of all firms in a given cell - the larger the dot and the brighter the color, the larger the market share or the larger the Bang of firms in the given percentile.

Bang. The larger (warmer colors of) the dots, the stronger the relative Bang of that group of firms compared to the average business. This panel, therefore, illustrates the key trade-off in the design of R&D subsidies: firms that are most responsive (high ϵ) and are characterized by the largest Bang (large, warm-colored dots) are also the most expensive to support (high values of r). The next subsection turns to analyzing these trade-offs more systematically.

5.2 Relative Bangs for the Buck

We now move to analyzing the potential for R&D policies targeted at different groups of incumbent firms. We do so by focusing on the *relative* Bang for the Buck which measures how much more (or less) Bang for the Buck a policy maker could get if they targeted only firms in a particular group k, relative to simply handing out the subsidy indiscriminately to all businesses.

The first three columns of Table 7 report the relative Bang for the Buck, the relative Bang and the relative Buck. The rows indicate these statistics for selected groups of businesses and for all firms together. The remainder of the table reports the components which drive the Bang and the Buck: (i) market shares, m_k, (ii) average micro-elasticities, $\bar{\epsilon}_k^B$ ($\bar{\epsilon}_k$), (iii) average sales-weighted growth rates, g_k, (iv) R&D shares, r_k, (v) heterogeneity in growth, θ_k^g and (vi) heterogeneity in R&D, θ_k^s.[32]

[32]Recall from Section 3.2 that due to the presence of negative growth rates among some businesses,

Before diving into the results, let us note that our decompositions in Proposition 2 offer intuitive ways of understanding the drivers of the relative Bang for the Buck. First, if firm group k is characterized by growth rates (g_k), micro-elasticities ($\bar{\epsilon}_k^B$) and heterogeneity (θ^g) which are identical to those in the economy as a whole, then $B_k/B = m_k$ and $C_k/C = r_k$. Therefore, comparing the relative Bang and Buck to the respective sales and R&D shares reveals the extent to which a particular firm group "punches above its weight" in terms of the Bang and Buck. For the same reason, if firm group k is characterized by R&D shares larger than sales shares ($r_k > m_k$), then it is a relatively expensive policy target. This is because their weight in aggregate growth is lower than their exposure to the policy. In what follows, we refer back to these comparisons when analyzing each group of firms in turn.

R&D intensive firms. The first row of Table 7 shows that focusing innovation subsidies on R&D intensive firms provides a *lower* Bang for the Buck than when simply handing out the subsidy indiscriminately to all businesses, i.e. $A_{RD-int.}/A = 0.94$. Looking at the remaining columns reveals the reasons behind this.

On the one hand, compared to the average firm, R&D intensive businesses are twice as responsive to changes in R&D subsidies ($\bar{\epsilon}_{RD-int.}^B = 0.12$ vs $\bar{\epsilon}_{all}^B = 0.06$) and they grow somewhat faster ($g_{RD-int.} = 0.06$ vs $g_{all} = 0.04$). These features are favorable in terms of the Bang.

On the other hand, however, R&D intensive firms account for over 3/4 of *all* R&D expenditures. This out-sized "exposure" to innovation subsidies makes them an expensive policy target. Put together, while R&D intensive firms have the potential to contribute substantially to aggregate growth, they do so at a relatively high cost.

Small firms. The second row of Table 7 shows that the Bang for the Buck of small businesses is about 1/3 larger compared to that of all firms together. At face value, this may be taken as evidence supportive of existing R&D schemes aimed at small and medium sized enterprises. We revisit this important point below.

Looking at the underlying drivers reveals that the strong relative Bang for the Buck of small firms is predominantly driven by their high average growth ($g_{small} = 0.10$ vs $g_{all} = 0.04$). In fact, small firms punch considerably above their weight as their relative Bang is three times greater than their market share ($B_{small}/B = 0.09$ vs $m_{small} = 0.03$).

The above positive effects are somewhat dampened by the fact that small firms are relatively expensive as a policy target—they account for more R&D spending then they

we separately report average micro-elasticities entering the Bang, $\bar{\epsilon}_k^B$ and those used in the Buck, $\bar{\epsilon}_k$. Intuitively, the discrepancy between these two disappears for "gazelles" which feature only positive growth rates, i.e. $\bar{\epsilon}_k^B = \bar{\epsilon}_k$. Note further that due to rounding of each Bang and Buck component, the products of the respective components do not exactly deliver the relative numbers reported in the first three columns. Finally, Appendix F.2 also considers asymptotic and bootstrapped standard errors for our relative Bangs for the Buck.

Table 7: Relative Bang for the Buck

Firm group	Relative Bang & Buck			Bang Components				Buck Components		
	A_k/A	B_k/B	C_k/C	m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.94	0.78	0.83	0.35	0.12	0.06	4.36	0.78	0.36	1.01
Small	1.39	0.09	0.06	0.03	0.08	0.10	6.50	0.05	0.26	2.03
Young	2.09	0.13	0.06	0.05	0.17	0.11	1.94	0.05	0.26	1.83
Gazelles	2.68	0.47	0.17	0.08	0.31	0.40	0.76	0.14	0.31	1.61
All	1.00	1.00	1.00	1.00	0.06	0.04	6.69	1.00	0.21	1.50

Note: The first three columns report the relative Bang for the Buck (A_k/A), relative Bang (B_k/B) and relative Buck (C_k/C). The next four columns show the drivers of the Bang—sales shares (m_k), micro-elasticities ($\bar{\epsilon}_k^B$), size-weighted growth (g_k) and growth heterogeneity (θ_k^g). The last three columns report the drivers of the Buck—R&D shares (r_k), micro-elasticities ($\bar{\epsilon}_k$) and R&D heterogeneity (θ_k^s). As explained in the main text, the difference between $\bar{\epsilon}_k^B$ and $\bar{\epsilon}_k$ is driven by the presence of firms with negative growth rates. The rows report values for different groups of firms as defined in the main text. The final row provides values for all firms as a whole.

do for sales ($r_{small} = 0.05$ vs $m_{small} = 0.03$). Overall, however, small firms are more efficient in generating a Bang than they are costly in terms of their Buck.

Young firms. The previous paragraphs suggested that, relative to subsidizing all businesses indiscriminately, small firms are a more suitable target in terms of their Bang for the Buck. However, in what follows, we show that young businesses fare even better. More importantly, we document that the Bang and Buck prowess of small firms is in fact predominantly driven by their relatively young age.

In particular, businesses which are less than 6 years since their IPO turn out to generate a Bang for the Buck which is more than twice as large as that of all firms. Inspecting the underlying drivers reveals that high growth ($g_{young} = 0.11$ vs $g_{all} = 0.04$) and a high average micro-elasticity ($\bar{\epsilon}_{young}^B = 0.17$ vs $\bar{\epsilon}_{all}^B = 0.06$) are the main sources of this strength.

Since size and age are closely related in the data (correlation coefficient of log sales and log age of 0.43), it is important for policy makers to understand which characteristic is the better indicator of a strong relative Bang for the Buck. To investigate this, we separate the group of small firms into small-young (small businesses which are less than 6 years from their respective IPOs) and small-old (all other small firms).

As we show in more detail in Appendix F.1, the strong relative Bang for the Buck of small firms is entirely driven by small-young businesses. In fact, small-old firms are characterized by a relative Bang for the Buck which is effectively identical to a uniform subsidy, $A_{small-old}/A = 1.02$.

Therefore, according to our framework, age is the more direct indicator of a strong Bang for the Buck and would be a better dimension on which to base R&D policies. This finding parallels with existing research on the importance of distinguishing firm age and size when analyzing job creation and productivity growth (see e.g. Haltiwanger et al., 2013). We contribute to this debate by pointing out that young businesses are also important propagators of R&D policies.

Gazelles. Finally, the most promising group of firms in terms of their relative Bang for the Buck are gazelles. The key reasons for their strong performance are a high average micro-elasticity ($\bar{\epsilon}^B_{gazelles} = 0.31$ vs $\bar{\epsilon}^B_{all} = 0.06$) and—by construction—very fast growth rates ($g_{gazelles} = 0.40$ vs $g_{all} = 0.04$). These two forces outweigh the fact that gazelles are relatively expensive to subsidize (they account for more R&D than they do for sales, $r_{gazelles} = 0.14$ vs $m_{gazelles} = 0.08$) and that they are characterized by "unfavorable" heterogeneity. The latter can be seen from the fact that $\theta^g_{gazelles} < 1$, indicating that large/fast-growing gazelles typically have lower micro-elasticities.

From the considered group of firms, gazelles are by far the most efficient in generating an aggregate Bang for the Buck—almost three times that of an indiscriminate subsidy of all firms. In what follows, we discuss how the persistence of firm growth may affect our conclusions.

5.3 Spillovers and Dynamics

As a final step in our analysis, we turn to investigating the role of technological spillovers and dynamics. The former may be particularly important for R&D intensive firms which account for the vast majority of R&D expenditures. Ignoring possible spillovers may under-estimate their relative Bang for the Buck. In contrast, the latter may be particularly important when gauging the impact of young and fast-growing firms. Ignoring the possibly temporary nature of firm growth may over-estimate their relative Bang for the Buck.

Spillovers. In order to quantify how spillovers affect our results, we must first make a stand on how important spillovers are for firm-level growth on average. Existing research suggests that estimating technological spillovers across firms is an imprecise endeavor which is sensitive to model specifications, levels of aggregation or the particular empirical measure of spillovers (see e.g. Hall et al., 2010, for a survey). Therefore, in what follows we lean on recent estimates (see e.g. Matray, 2021; Dyevre, 2024) and offer a range of possible values for the importance of spillovers for firms' growth.

Table 8 shows the relative Bang for the Buck which accounts for spillovers. In addition, building on Proposition 4, we also decompose these values into the separate contributions of "own" R&D, "internal" spillovers within the considered group of firms and "external" spillovers which provide a boost for firms outside the considered group of businesses. Note that when considering a uniform subsidy to all firms, external spillovers are zero by

Table 8: Relative Bang for the Buck including spillovers

Firm group	$\frac{A_k}{A}$	Accounting for Spillovers $\frac{A_k^{spill}}{A^{spill}}$	$\frac{B_k^{int}}{B_k}$	$\frac{B_k^{ext}}{B_k}$	$\frac{A_k^{spill}}{A^{spill}}$	$\frac{B_k^{int}}{B_k}$	$\frac{B_k^{ext}}{B_k}$
		$\omega = 0.85$			$\omega = 0.75$		
R&D-int.	0.94	0.97	0.09	0.11	0.98	0.16	0.18
Small	1.39	1.19	0.00	0.02	1.06	0.00	0.04
Young	2.09	1.84	0.01	0.04	1.67	0.01	0.07
Gazelles	2.68	2.38	0.02	0.04	2.18	0.03	0.06
All	1.00	1.00	0.17	0.00	1.00	0.29	0.00

The table presents the Bang for the Buck and its components when we account for technological spillovers between firms. We use the sample of firms that we use for Table 7. A_k^{spill} and A_k correspond to Bang for the Buck with and without the spillovers, respectively. The components are, in turn, $B_k^{int} = \sum_{i\in\Omega_k} m_i g_i \sum_{j\in\Omega_k|\neq i} \sigma_{i,j}\epsilon_j$ and $B_k^{ext} = \sum_{j\in\Omega_{\neq k}} m_j g_j \sum_{i\in\Omega_k} \sigma_{j,i}\epsilon_i$. B_k marks the Bang without spillovers. The entries in the first two columns are expressed relative to the outcomes under a uniform subsidy.

construction.

In particular, the first two columns of Table 8 report the relative Bang for the Buck with and without accounting for spillovers. As can be seen, R&D intensive businesses are the only firm group which features a stronger relative Bang for the Buck once spillovers are taken into account. This reflects our initial conjecture that R&D intensive firms may have positive growth effects stretching to other businesses.

Indeed, columns B_k^{int}/B_k and B_k^{ext}/B_k show that, compared to other groups of businesses, R&D intensive firms are characterized by relatively strong internal and especially external spillovers. That said, however, the magnitude of these effects does not alter our ranking of firm groups. Young firms and gazelles remain to be two groups of firms with the highest relative Bang for the Buck, even when accounting for spillovers across firms.

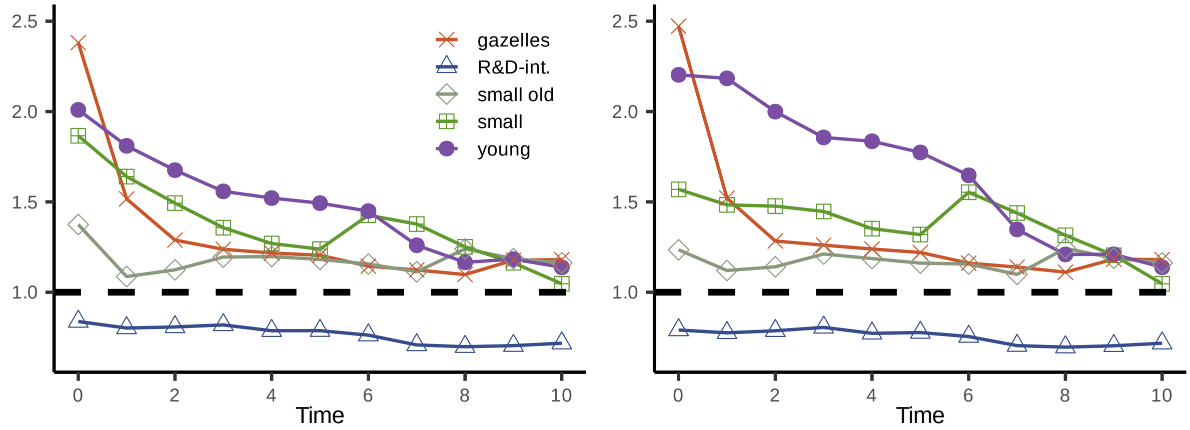
Dynamics. Our baseline results provide the average effects across all firms and time periods. However, from the standpoint of the policy implementation, it is crucial to understand how the Bang for the Buck at the firm level changes over time. Are yesteryear's gazelles still the most cost-effective firms to support?

To account for possible changes in the relative Bangs for the Buck, we classify firms into our categories in the same fashion as described in Section 3.4 using annual data. Then, keeping this classification fixed, we compute the *annual* relative Bang for the Buck for each group of firms over the following years and present the results in Figure 3. While the left panel (a) shows results for an unbalanced panel, the right panel (b) does the same for a balanced panel of firms. That is, we focus on firms that continuously report balance sheet information for at least 10 years.

Focusing on the unbalanced set of firms (left panel), the results show that relative

Figure 3: Relative Bangs for the Buck: Dynamics

(a) Relative Bangs for the Buck, Unbalanced Panel (b) Relative Bangs for the Buck, Balanced Panel



Note: The figure reports relative Bangs for the Buck across our firm groups. The classification into groups is made at $t = 0$. Then, we evaluate the changes in the Bangs and the Buck for a fixed classification. For example, the point on the purple line line with crossed-square markers at Time = 6 corresponds to the cost-effectiveness of supporting firms that were classified as young 6 years ago. Left panel (a) shows results for pooled data of all firms. The right panel (b) restricts sample to a balanced panel of firms that continuously report balance sheet information for at least 10 years.

Bangs for the Buck tend to converge (with the exception of those of R&D-intensive firms). However, young, and to a lesser extent fast-growing, firms retain their top ranking. As discussed previously, the performance of small firms is almost exclusively driven by the fact that many of them are young. Indeed, small-old firms have a relative Bang for the Buck which is persistently low. Similar patterns can be observed for the balanced panel, where conditioning on firm survival only exacerbates the prowess of young businesses.

The reason behind these patterns is that the Bang for the Buck does not depend on a single variable. Instead, it is determined by a combination of several drivers simultaneously (growth, size, R&D and micro-elasticities). Intuitively, as fast-growing firms expand quickly, they gain market share. Therefore, their weight in the economy remains relatively stable, even after their initially fast growth dissipates (see Appendix F.3 for further details).

5.4 Discussion

This paper develops an empirical framework for analyzing the aggregate impact of firm-level R&D incentives. The approach relies on fundamental economic trade-offs facing businesses when deciding on R&D investment and it offers measurable statistics for computing the marginal impact of changes in R&D policies on aggregate outcomes. In this section, we briefly discuss the robustness of our findings along several dimensions.

Financial Frictions. Our analytical framework is based on firms optimally balancing the marginal costs and benefits of innovation. R&D subsidies directly affect the latter, since they make innovation cheaper. However, if firms are credit constrained, then R&D subsidies may have an additional impact on firms' decisions through alleviating credit constraints.

In this sense, our framework can be viewed as one aimed at the set of *un-constrained*. Arguably, this is the relevant group of businesses given the strong empirical skewness of R&D towards large firms (see Table 5). Moreover, Ottonello and Winberry (2025) develop a model of innovation under financial frictions and use Compustat data to estimate that indeed "the majority of innovation at a given time is performed by unconstrained firms."

Nevertheless, in Appendix B.3 we extend our framework to allow for financial constraints which are alleviated as firms grow larger. In this setting, firm-level micro-elasticities are a combination of two factors. First, and as in our baseline analysis, they depend on R&D-to-profits. Second, because of borrowing constraints, they also depend on the elasticity of the shadow value of funds with respect to R&D subsidies, $\epsilon_{\lambda,\tau}$.

While $\epsilon_{\lambda,\tau}$ is generally unobserved, it is also likely to be dwarfed by firms' R&D to profit ratios. To understand this, note that the shadow value of funds simply equals one for unconstrained firms. In addition, Ottonello and Winberry (2020) estimate an average excess return on capital (a proxy for the shadow value of funds) of about 5%. Therefore, considering an extreme upper bound, where a marginal increase in R&D subsidies leads to complete *elimination* of all financing constraints, implies $\epsilon_{\lambda,\tau} \approx -0.05$. This extreme upper bound is only about 1/5 of the average R&D-to-profit ratio in the data.

Technology Spillovers vs Business Stealing. In our approach, we focus on technological spillovers between businesses, measured by technology-proximity and quality-adjusted weights, $\sigma_{i,j}$. At the same time, however, firms may be connected also in the "product space." If so, businesses may encounter "business stealing" from firms which have managed to innovate, rather than reaping positive technological spillovers.

Indeed, according to estimates in Bloom et al. (2013), both types of spillovers (technology and product) are present in the data. However, technology spillovers are quantitatively much more important. Moreover, R&D intensive firms—which account for the majority of R&D expenditures—are characterized by the lowest relative Bang for the Buck (see Table 7). For these reasons, we focus on the possible positive effects of technological spillovers, as allowing for business stealing would likely only further decrease the contribution of R&D-intensive firms.

General Equilibrium, the Distribution of Firms and Optimal Policies. The theoretical underpinning of our analysis focuses on the *marginal impact* of R&D incentives, holding the distribution of firms and aggregates (in particular growth) constant.

We make four comments in this regard.

First, survey evidence suggests that firm expectations indeed respond much more strongly to firm-specific conditions, rather than aggregate developments (see e.g. Born et al., 2024). In this sense, our micro-elasticities are likely capturing the dominant trade-offs for incumbent firms.

Second, in our framework aggregate growth affects future profits (through household demand and factor prices). To the extent that profits of different firms respond to aggregate conditions in the same way, then our conclusions about the *relative* Bang for the Buck remain unchanged even when accounting for general equilibrium effects.

Third, aside from their impact on incumbent firms—the topic of this paper—(non-marginal) changes in R&D incentives may also affect the distribution of firms via firm entry and selection effects (see e.g. Acemoglu et al., 2018).[33] Our framework can, therefore, be viewed as an approximation around the prevailing firm distribution and associated balanced growth path, similar to e.g. Atkeson and Burstein (2019).

Finally, in the absence of stronger assumptions, this paper does not consider optimal policies. Instead, the focus is on highlighting how firm heterogeneity shapes the effectiveness of R&D policies in stimulating aggregate growth and how such heterogeneity can be measured with sufficient statistics. We do, however, believe that studying firms' extensive margins—both in terms of the decision to start investing into R&D and in terms of firm entry—and considering whether our sufficient statistic approach can be extended (under additional assumptions) to study optimal R&D policies are fruitful avenues for future research.

6 Conclusion

In this paper, we develop a tractable framework for evaluating which groups of incumbent firms offer the biggest Bang for the Buck—boost to aggregate growth per dollar spent on the policy change. Our approach—grounded in modern endogenous growth models—allows us to use readily available data to measure the responsiveness of individual firms to changes in R&D incentives and to aggregate such responses. We validate our approach using a range of firm-level datasets and apply our framework to Compustat data. The results suggest that firm heterogeneity plays a key role in understanding the aggregate impact of firm-level R&D incentives. In addition, young and fast-growing firms offer the strongest Bang for the Buck and this conclusion is robust to considering knowledge spillovers and dynamics.

We believe that our framework opens the door to a range of additional interesting questions. For instance, are different types of R&D investment associated with different

[33]Note that recent empirical evidence suggests that firm-level innovation does not seem to respond to R&D incentives along the extensive margin (see e.g. Dechezlepretre et al., 2023).

Bangs for the Buck? Can we search for a well-defined group of firms with the highest Bang for the Buck which could represent a valid policy target? What is the impact of non-marginal changes in subsidies? How is firm entry and exit affected by such policy changes? We leave these and other open questions for future research.

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Online Appendix

Bang for the Buck: Aggregate Impact of Firm-Level R&D Incentives

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Contents

A Proofs

A.1 Proposition 1

For convenience, let us repeat the key definitions (under Assumption 1):

$$B = \frac{\mathrm{d}G}{\mathrm{d}\log\tau}, \tag{A1}$$

$$C = \frac{\mathrm{d}\log T}{\mathrm{d}\log\tau}, \tag{A2}$$

$$T = \tau\sum_i \widetilde{s}_i(x_i), \tag{A3}$$

$$S = (1-\tau)\sum_i \widetilde{s}_i(x_i), \tag{A4}$$

$$G = \sum_i m_i g_i, \tag{A5}$$

where we note that both the Bang, B, and the Buck, C, are defined as *impact* responses, holding firms' sales shares fixed. In this setting, the Bang is given by differentiating (A5) with respect to τ:

$$\begin{aligned} dG &= \sum_i m_i \frac{\partial g_i}{\partial\tau} d\tau = \sum_i m_i \epsilon_i \frac{g_i}{\tau} d\tau = \sum_i m_i g_i \epsilon_i \frac{d\tau}{\tau} \\ B &= \frac{dG}{d\tau/\tau} = \sum_i m_i g_i \epsilon_i, \end{aligned}$$

where we have used the definition of our micro-elasticity, $\epsilon_i = \frac{\partial x_i}{\partial\tau}\frac{\tau}{x_i} = \frac{\partial g_i}{\partial\tau}\frac{\tau}{g_i}$. Next, to obtain the Buck, we differentiate (A3) with respect to τ:

$$\begin{aligned} dT &= \sum_i \left(\overbrace{\widetilde{s}_i(x_i)}^{\text{direct effect}} + \overbrace{\tau \frac{\partial\widetilde{s}_i(x_i)}{\partial x_i}\frac{\partial x_i}{\partial\tau}}^{\text{indirect effect}} \right) d\tau = \sum_i \left(\widetilde{s}_i(x_i) + \tau\psi\epsilon_i \frac{\widetilde{s}_i(x_i)}{\tau} \right) d\tau \\ &= \frac{\tau}{1-\tau}\sum_i \overbrace{(1-\tau)\widetilde{s}_i(x_i)}^{=s_i(x_i)} (1+\psi\epsilon_i)\frac{d\tau}{\tau} = \overbrace{\frac{\tau}{1-\tau}S}^{=T} \sum_i \overbrace{\frac{s_i(x_i)}{S}}^{=r_i} (1+\psi\epsilon_i)\frac{d\tau}{\tau} \\ C &= \frac{dT/T}{d\tau/\tau} = \sum_i r_i (1+\psi\epsilon_i), \end{aligned}$$

where $r_i = s_i/S$ are firm-level R&D shares and where in the first line we use our definitions of the R&D cost elasticity, $\psi = \frac{\partial\widetilde{s}_i(x_i)}{\partial x_i}\frac{x_i}{\widetilde{s}_i(x_i)}$, and the micro-elasticity, ϵ_i. In the second line, we use $s_i(x_i) = (1-\tau)\widetilde{s}_i(x_i)$ and the fact that combining (A4) and (A3) gives $T = \tau/(1-\tau)S$.

A.2 Proposition 2

Recall that the Bang and the Buck are given by:

$$
\begin{aligned}
B &= \sum_i m_i g_i \epsilon_i, \\
C &= \sum_i r_i \left(1 + \psi \epsilon_i\right),
\end{aligned}
$$

where $m_i = y_i/Y$ and $r_i = s_i/S$. We can now rewrite the Bang as

$$
B = \frac{1}{Y}\sum_i y_i g_i \epsilon_i = \frac{N}{Y}\frac{1}{N}\sum_i g_i^y \epsilon_i = \frac{1}{\overline{y}}\left(\overline{g}^y \overline{\epsilon} + \mathrm{cov}(g^y, \epsilon)\right)
$$

where $g_i^y = y_i g_i$ and where we have used the fact that for two variables x and y, $\frac{1}{N}\sum_{i=1}^N x_i y_i = \overline{x}\,\overline{y} + \mathrm{cov}(x, y)$. Next, denoting $n_k = N_k/N$ and $\overline{y} = Y/N$, we use the same logic for group-specific Bangs:

$$
\begin{aligned}
B_k =& \frac{1}{Y}\sum_{i\in\Omega_k} y_i g_i \epsilon_i = \frac{N}{Y}\frac{N_k}{N}\frac{1}{N_k}\sum_{i\in\Omega_k} g_i^y \epsilon_i = \frac{n_k}{\overline{y}}\left(\overline{g}_k^y \overline{\epsilon}_k + \mathrm{cov}(g_k^y, \epsilon_k)\right) \\
=& \frac{n_k}{\overline{y}}\overline{g}_k^y \overline{\epsilon}_k \left(1 + \frac{\mathrm{cov}(g_k^y, \epsilon_k)}{\overline{g}_k^y \overline{\epsilon}_k}\right) = \frac{N}{Y}\frac{N_k}{N}\frac{1}{N_k}\sum_{i\in\Omega_k} y_i g_i \overline{\epsilon}_k \left(1 + \frac{\mathrm{cov}(g_k^y, \epsilon_k)}{\overline{g}_k^y \overline{\epsilon}_k}\right) \\
=& \frac{Y_k}{Y}\sum_{i\in\Omega_k} \frac{y_i}{Y_k} g_i \overline{\epsilon}_k \left(1 + \frac{\mathrm{cov}(g_k^y, \epsilon_k)}{\overline{g}_k^y \overline{\epsilon}_k}\right) = m_k g_k \overline{\epsilon}_k \theta_k^g.
\end{aligned}
$$

From the above, we see that $B = \sum_k g_k m_k \overline{\epsilon}_k \theta_k^g$. Using the same logic, we can rewrite the Buck as

$$
C = 1 + \frac{\psi}{S}\sum_i s_i \epsilon_i = 1 + \frac{N\psi}{S}\frac{1}{N}\sum_i s_i \epsilon_i = 1 + \psi\left(\overline{\epsilon} + \frac{\mathrm{cov}(s, \epsilon)}{\overline{s}}\right),
$$

where $\overline{s} = S/N$ is average R&D expenditure. Extending the above to group-specific Bucks, we can write

$$
\begin{aligned}
C_k =& \frac{1}{S}\sum_{i\in\Omega_k} s_i \left(1 + \psi \epsilon_i\right) = \frac{\sum_{i\in\Omega_k} s_i}{S} + \psi \frac{N}{S}\frac{N_k}{N}\frac{1}{N_k}\sum_{i\in\Omega_k} s_i \epsilon_i = r_k + \frac{n_k}{\overline{s}}\psi\left(\overline{s}_k \overline{\epsilon}_k + \mathrm{cov}(s_k, \epsilon_k)\right) \\
=& r_k + \frac{n_k}{\overline{s}}\psi \overline{s}_k \overline{\epsilon}_k \left(1 + \frac{\mathrm{cov}(s_k, \epsilon_k)}{\overline{s}_k \overline{\epsilon}_k}\right) = r_k + \psi \frac{N}{S}\frac{N_k}{N}\frac{1}{N_k}\sum_{i\in\Omega_k} s_i \overline{\epsilon}_k \left(1 + \frac{\mathrm{cov}(s_k, \epsilon_k)}{\overline{s}_k \overline{\epsilon}_k}\right) \\
=& r_k + \psi \frac{S_k}{S}\overline{\epsilon}_k \left(1 + \frac{\mathrm{cov}(s_k, \epsilon_k)}{\overline{s}_k \overline{\epsilon}_k}\right) = r_k(1 + \psi \overline{\epsilon}_k \theta_k^s).
\end{aligned}
$$

From the above, we see that $C = \sum_k r_k(1 + \psi \overline{\epsilon}_k \theta_k^s)$.

A.3 Proposition 3

Recall that firm-level output and profits can be written as:

$$\begin{aligned} q_{i,t+1} =& q_{i,t}(1+g_{i,t}), \\ \pi_{i,t} =& \pi^o_{i,t} - s_i(x_{i,t}), \end{aligned}$$

where $\pi^o_{i,t}$ are "operating profits" which are (by construction) independent of R&D subsidies. Further, note the following relationships hold in this environment:

$$\begin{aligned} \epsilon_{q'q} =& \frac{\partial q_{i,t+1}}{\partial q_t}\frac{q_t}{q_{t+1}} = 1, \\ \epsilon_{\pi q} =& \frac{\partial \pi_{i,t}}{\partial q_{i,t}}\frac{q_{i,t}}{\pi_{i,t}} = \epsilon_{\pi^o q}\left(1 - \frac{s_{i,t}}{\pi_{i,t}}\right) + \epsilon_{sq}\frac{s_{i,t}}{\pi_{i,t}}, \end{aligned}$$

where $\epsilon_{\pi^o q} = \frac{\partial \pi^o_{i,t}}{\partial q_{i,t}}\frac{q_{i,t}}{\pi^o_{i,t}}$ and $\epsilon_{sq} = \frac{\partial s_{i,t}}{\partial q_{i,t}}\frac{q_{i,t}}{s_{i,t}}$. Note that both $\epsilon_{\pi^o q}$ and ϵ_{sq} are independent of τ. The former by construction, the latter due to the log-linear nature of how subsidies enter R&D expenditures.

Suppressing the firm-specific index i to lighten the notation, firms optimally choose R&D expenditures in order to balance the costs and benefits of growth:

$$v(q_t) = \max_{g_t} \sum_{j=0}^{\infty} \beta^j \pi(q_{t+j}),$$

where β^j is a discount factor (possibly reflecting firm exit). Optimal firm-level growth rates then satisfy:

$$\begin{aligned} \psi\frac{s(x_t)}{x_t} =& \sum_{j=1} \beta^j \frac{\partial \pi_{t+j}}{\partial q_{t+j}} \Pi_{k=1}^{j-1} \frac{\partial q_{t+k+1}}{\partial q_{t+k}} \frac{\partial q_{t+1}}{\partial g_t} = \sum_{j=1} \beta^j \epsilon_{\pi q,t+j} \frac{\pi_{t+j}}{q_{t+j}} \Pi_{k=1}^{j-1} \epsilon_{q'q} \frac{q_{t+k+1}}{q_{t+k}} q_t \\ \underbrace{\psi\frac{s(x_t)}{x_t}}_{\text{marginal cost, MC}} =& \underbrace{\sum_{j=1} \beta^j \epsilon_{\pi q,t+j} \frac{\pi_{t+j}}{1+g_t}}_{\text{marginal benefit, MB}} \end{aligned} \tag{A6}$$

Finally, to obtain our micro-elasticity, we use the fact that for marginal changes, $d(1-\tau) = -d\tau$ and we differentiate (A6) with respect to $1-\tau$. In doing so, we note that $\epsilon_{\pi q}$ is independent of τ and that the indirect effects of τ (operating via changes in *future* growth rates, g_{t+j} for $j > 0$) exactly offset each other thanks to the envelope condition (a lower price of R&D raises firms' growth rates, but at the optimum the associated marginal R&D costs exactly offset the associated marginal profits).

Therefore, using Assumption 2, what we are left with is given by:

$$\frac{\psi}{g_t}\left(\frac{\partial s(x_t)}{\partial x_t}\frac{\partial x_t}{\partial 1-\tau}-\frac{s(x_t)}{x_t}\frac{\partial x_t}{\partial 1-\tau}\right)d(1-\tau)=\sum_{j=1}\beta^j\frac{\epsilon_{\pi q,t+j}}{1+g_t}\frac{\partial \pi_{t+j}}{\partial 1-\tau}d(1-\tau)$$

$$\frac{\psi}{x_t}\left(-\psi\epsilon\frac{s(x_t)}{x_t}\frac{x_t}{1-\tau}+\epsilon\frac{s(x_t)}{x_t}\frac{x_t}{1-\tau}\right)d(1-\tau)=-\sum_{j=1}\beta^j\frac{\epsilon_{\pi q,t+j}}{1+g_t}\pi_{t+j}\frac{s_{t+j}}{\pi_{t+j}}\frac{d(1-\tau)}{1-\tau}$$

$$\underbrace{\frac{\psi s(x_t)}{x_t}}_{MC}(\psi-1)\,\epsilon\frac{d(1-\tau)}{1-\tau}=\underbrace{\sum_{j=1}\beta^j\frac{\epsilon_{\pi q,t+j}}{1+g_t}\pi_{t+j}}_{MB}\frac{s}{\pi}\frac{d(1-\tau)}{1-\tau}$$

$$\epsilon=\frac{s}{\pi}\frac{1}{\psi-1}$$

A.4 Proposition 4

Let us begin by repeating how a firm's growth depends on its "own" R&D efforts and on "external" spillovers:

$$g_i=\eta_{own}x_i+\eta_{ext}\underbrace{\sum_{j\neq i}\alpha_{i,j}x_j}_{S_i^{ext}}\,.$$

In this environment, changing subsidies affects firms' own incentives to conduct R&D, but it also creates spillover effects from increased R&D of other firms:

$$\begin{aligned}
\epsilon_i^{spill}&=\frac{\partial g_i}{\partial\tau}\frac{\tau}{g_i}=\left[\eta_{own}\frac{\partial x_i}{\partial\tau}+\eta_{ext}\sum_j\alpha_{i,j}\frac{\partial x_j}{\partial\tau}\right]\frac{\tau}{g_i}\\
&=\left[\eta_{own}\frac{x_i}{\tau}\epsilon_i+\eta_{ext}\sum_j\alpha_{i,j}\epsilon_j\frac{x_j}{\tau}\right]\frac{\tau}{g_i}\\
&=\underbrace{\eta_{own}\frac{x_i}{g_i}}_{\omega_i}\epsilon_i+\underbrace{\eta_{ext}\frac{S_i^{ext}}{g_i}}_{1-\omega_i}\sum_j\underbrace{\frac{\alpha_{i,j}x_j}{S_i^{ext}}}_{\sigma_{i,j}}\epsilon_j=\omega_i\epsilon_i+(1-\omega_i)\sum_j\sigma_{i,j}\epsilon_j,
\end{aligned}$$

A.5 Proposition 5

Let us consider that only group k of firms is subsidized and that spillover effects exist. In this case, we can write the group-specific Bang as:

$$B_k^{spill}=\overbrace{\sum_{i\in\Omega_k}m_ig_i\omega_i\epsilon_i}^{\text{“own” Bang, }B_k^{own}}+\overbrace{\sum_{i\in\Omega_k}m_ig_i(1-\omega_i)\sum_{j\in\Omega_k|\neq i}\sigma_{i,j}\epsilon_j}^{\text{“internal” spillover Bang, }B_k^{int}}+\overbrace{\sum_{j\in\Omega_{\neq k}}m_jg_j(1-\omega_j)\sum_{i\in\Omega_k}\sigma_{j,i}\epsilon_i}^{\text{“external” spillover Bang, }B_k^{ext}}$$

Note that the group-specific Buck is the same as without spillovers. Next, we can write the group-specific Bang for the Buck with spillovers as follows:

$$
\begin{aligned}
A_k^{spill} =& \frac{B_k^{spill}}{C_k} = \frac{B_k}{C_k}\left(\frac{B_k^{own}}{B_k} + \frac{B_k^{int}}{B_k} + \frac{B_k^{ext}}{B_k}\right) \\
=& A_k\left(\frac{B_k^{own}}{B_k} + \frac{B_k^{int} + B_k^{ext}}{B_k}\right)
\end{aligned}
\tag{A7}
$$

B Additional Analytical Results

In this Appendix, we describe an example of an entire structural model consistent with our framework in the main text and provide an extension to Proposition 3.

B.1 Proposition 3: Workhorse Model

We now present an example of a full structural model that is consistent with Assumptions 1 and 2 in the main text. Time is discrete and we use primes to denote next period values.

Production. We assume that there is a continuum of individual firms, indexed by i, each producing a differentiated final consumption good, q_i. The final goods are consumed by the representative household endowed with constant elasticity preferences of over consumption bundle,

$$
U = \sum_{t=0}^{\infty} \beta^t u(C_t), \qquad C = \left[\int_i q_i^{\frac{\eta-1}{\eta}} \,\mathrm{d}i\right]^{\frac{\eta}{\eta-1}},
$$

where $\eta > 1$ is the elasticity of substitution between goods. The household faces aggregated budget constraint

$$
Z' + \int_i c_i p_i \,\mathrm{d}j = WN + (1+R)Z,
$$

where Z marks the holdings of diversified equity portfolio of all firms in the economy, p_i is the firm-specific goods price (relative to the aggregate price index P which is normalized to 1), W is the aggregate wage and N is the aggregate labor supply. Without loss of generality, we assume that the household supplies inelastically one unit of labor.

The optimal consumption choice implies that each firm faces downward sloping demand for their product:

$$
q_i = p_i^{-\eta} Y, \tag{A8}
$$

and Y is aggregate expenditure. Intermediate goods are produced using a linear technology combining production labor, n_i, and a firm-specific (endogenous) productivity,

a_i:

$$q_i = a_i n_i. \tag{A9}$$

Innovation. Firms strive to improve their own productivity by investing resources into R&D. Firms invest s_i units of labor into R&D in return for a probability x_i with which they successfully improve upon their current productivity level. We assume that R&D costs are convex in the innovation probability, x:

$$s_i = \overline{s}_i x_i^{\psi}, \tag{A10}$$

where we assume that $\psi = 2$ and that $\overline{s}_i > 0$ is a potentially firm-specific and time-varying scaling factor described below. If successful, innovations lead to an increase in firm-level productivity by a factor of $(1 + \lambda_i)$, where $\lambda_i > 0$ is a potentially firm-specific constant:

$$a_i' = \begin{cases} a_i(1+\lambda_i) & \text{with probability } x_i \\ a_i & \text{with probability } 1 - x_i. \end{cases} \tag{A11}$$

Optimality conditions. Let us now describe the optimality conditions characterizing firms' choice of the price, production employees and R&D employees, $p(a_i)$, $n(a_i)$ and $s(a_i)$, respectively, as a function of firms' productivity a_i. The current profit is given by

$$\pi(a_i) = p(a_i)^{1-\eta} Y - W n(a_i) - W s(a_i),$$

Let β_i be firm-specific discount factor that includes also exogenous exit probability. The outside option of exiting firms is normalized to zero. Exiters are replaced by entrants who draw initial productivity from the distribution of incumbent firms. The firm value function becomes

$$V_i(a_i) = \pi(a_i) + \beta_i \left[x_i V_i(a_i(1+\lambda_i)) + (1-x_i) V_i(a_i), \right]$$

where we highlight that firm value functions are allowed to be firm-specific due to the heterogeneity in the deep parameters governing the technology and discount factors.

The optimal pricing choice is standard and implies a constant markup over the marginal cost, $p(a_i) = \frac{\eta}{\eta-1} \frac{W}{a_i}$. Consequently, the production labor demand is $n(a_i) = p_i^{-\eta} Y / a_i = \left(\frac{\eta}{\eta-1}\right)^{-\eta} a_i^{\eta-1} W^{-\eta} Y$. Finally, the optimal R&D labor satisfies

$$\psi W \overline{s}_i x_i^{\psi-1} = \beta_i \left(V(a_i(1+\lambda_i)) - V(a_i) \right)$$

Equilibrium. Let $Q = \left[\int a_i^{\eta-1} \, \mathrm{d}j \right]^{\frac{1}{\eta-1}}$ be the aggregate productivity index. We restrict attention to balanced growth path equilibria in which all aggregate variables grow at the same rate as Q. In what follows, let us denote stationarized variables by hats, for example $\widehat{a} = a/Q$ marks stationarized productivity.

Now, we introduce key assumption that specifies the shape of the R&D cost function $\overline{s}_i \equiv \rho_i \left(\widehat{a}_i\right)^{\eta-1}$, where ρ_i is a potentially firm specific, but time-invariant constant. Below, we show that this is the only functional form consistent with an existence of a balanced growth path equilibrium (BGP). Before that, let us denote the stationarized profits as

$$\widehat{\pi}(\widehat{a}_i) = \frac{\pi(a_i)}{Q} = \widehat{a}_i^{\eta-1}\widehat{W}^{1-\eta}\widehat{Y}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\frac{1}{\eta-1} - \widehat{W}\rho_i\widehat{a}_i^{\eta-1}x_i^2$$

The optimality conditions specified in the previous section imply that the firm value function is proportional to $\widehat{a}_i^{\eta-1}$. We prove this claim using a guess and verify method. Guess that the value function satisfies $V \equiv \widehat{v}_i\widehat{a}_i^{\eta-1}$ for a potentially firm-specific but time invariant constant $\widehat{v}_i > 0$. Given this guess, the optimal R&D policy implies that (using $\psi_r = 2$)

$$x_i^* = \frac{\beta_i}{2\widehat{W}(1+g)}\widehat{v}_i\left((1+\lambda_i)^{\eta-1} - 1\right)$$

The resulting $x_{r,i}$ is firm-specific, but—crucially—time invariant as it does not depend on the productivity $\widehat{a}_i$. With these intermediate results, the definition of firm value implies

$$\widehat{v}_i\widehat{a}_i^{\eta-1} = \widehat{a}_i^{\eta-1}\left[\widehat{W}^{1-\eta}\widehat{Y}\mu^* - \widehat{W}\rho_i(x_i^*)^2 + \widehat{v}_i\left(x_i^*(1+\lambda_i)^{\eta-1} + 1 - x_i^*\right)\right] \tag{A12}$$

where $\mu^* = \left(\frac{\eta}{\eta-1}\right)^{-\eta}\frac{1}{\eta-1}$ and where we used our guess on the functional form of the value function which we subsequently verified.

Along a BGP equilibrium, all aggregate variables grow at the rate $1+g$ equal to the growth rate of aggregate productivity index $Q = \left[\int a_i^{\eta-1}\,\mathrm{d}j\right]^{\frac{1}{\eta-1}}$. To verify this observe that since all costs are denominated in labor units and aggregate price index is normalized to unity, the aggregate resource constraint reads $C = Y$. Note further that the aggregate consumption can be written as $C = \left[\int_i \left(\left(\frac{\eta}{\eta-1}\right)^{-\eta} a_i^{\eta-1}W^{-\eta}C\right)^{\frac{\eta-1}{\eta}}\mathrm{d}j\right]^{\frac{\eta}{\eta-1}}$, which implies that $W = \frac{\eta-1}{\eta}\left[\int a_i^{\eta-1}\,\mathrm{d}j\right]^{\frac{1}{\eta-1}} = \frac{\eta-1}{\eta}Q$. Consequently, $\widehat{W} = \frac{\eta-1}{\eta}$.

Next, the labor market clearing implies that $N = 1 = \int n(a_i)\,\mathrm{d}j + \int_i r(a_i)$. Using the optimal policy derived above, we can show that $\int n(a_i)\,\mathrm{d}j = CQ^{-1}$. Furthermore, $\int_i r(a_i)\,\mathrm{d}j = \int \rho_i\left(\frac{a_i}{Q}\right)^{\eta-1}(x_{r,i}^*)^2$ is stationary. Therefore, the labor demand is constant along the balanced growth path.

Let us note that the labor market clearing requires that the labor devoted to R&D does not grow in a balanced growth equilibrium. This is only feasible if the mass of researchers required to deliver a given innovation probability grows at the same rate as the market size. Otherwise, as firms' profits increase, firms would spend increasing amounts of resources on R&D which violates labor market clearing in the limit. Therefore, the postulated scaling of R&D cost is necessary condition for the existence of the BGP.

The model satisfies Assumptions 1 and 2. Finally, we show that the stylized model above satisfies our key assumptions. As for Assumption 1, observe that the expected firm level growth rate is

$$1 + g = \frac{(a_i')^{\eta-1} - a_i^{\eta-1}}{a_i^{\eta-1}} = 1 + x_i^*[(1+\lambda_i)^{\eta-1} - 1].$$

Therefore, $\log g = \log x_i^* + \log[(1+\lambda_i)^{\eta-1} - 1]$. With these intermediate results we have

$$\log s_i = \log \rho_i + \log (\widehat{a}_i)^{\eta-1} + \psi(\log g - \log[(1+\lambda_i)^{\eta-1} - 1])$$

Consequently, $\frac{\mathrm{d}\log s_i}{\log g} = \psi$ is common across firms and time invariant as required by Assumption 1.

As for Assumption 2, it follows that

$$\frac{s_i}{\pi(a_i)} = \frac{s_i/Q}{\pi(a_i)/Q} = \frac{\widehat{W}\rho_i x_i^2}{\widehat{W}^{1-\eta}\widehat{Y}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\frac{1}{\eta-1} - \widehat{W}\rho_i x_i^2}.$$

The above expression is independent of firm's idiosyncratic state a_i. Furthermore, all variables on the right-hand side are constant along the balanced growth path. Consequently, while firm specific, the ratio of R&D expenses to profits is time invariant as required by Assumption 2.

B.2 Proposition 3: Extensions

In this section, we discuss the extension in which the firm profits follow a stochastic process, rather than deterministic path as in the baseline framework in the main text. As before, we assume that firm profits follow an additive specification in operating profits $\widetilde{\pi}_{i,t} = \widetilde{\pi}^o_{i,t} - s_i(x_{i,t})$. Here $\widetilde{\pi}^o_{i,t}$ denotes a realization of an i.i.d. random variable with a finite first moment. Any such random variable can always be represented by $\widetilde{\pi}^o_{i,t} = \pi^o_{i,t} + \zeta_{i,t}$ where $\pi^o_{i,t}$ is the mean of the process and $\zeta_{i,t}$ is a zero-mean, i.i.d. stochastic process. Therefore, we can represent the stochastic process for profits $\widetilde{\pi}$ as the sum of deterministic component, π, and stochastic component ζ.

Consider a firm that maximizes expected present discounted value of all future profits:

$$v_i(q_{i,t}) = \max_{g_{i,t}} \mathbb{E}_t \sum_{j=0} \beta_i^j \widetilde{\pi}_i(q_{i,t+j})$$

where $\mathbb{E}_t$ marks the expectation conditional on period-t information set. Next, we state a stochastic version of Assumption 2

ASSUMPTION **3.** *Assume that at the firm level, the operating profit is an i.i.d. stochastic process such that* $\mathbb{E}_t\left[\frac{s_{i,t}}{\pi_{i,t}} \mid X_{i,t-j}\right] = \mathbb{E}_t \frac{s_{i,t}}{\pi_{i,t}} = \frac{s_i}{\pi_i}$ *for all i, all t and all $j = 0, 1, \ldots, t$. Here, $X_{i,t-j}$ a generic idiosyncratic or aggregate variable in the firm problem.*

Assume further that $\frac{\mathrm{d}s}{\mathrm{d}\zeta} = 0$*, that is, the R&D cost are independent of the realization of profitability shocks.*

Note that Assumption 3 implies that

$$\max_{g_{i,t}} \mathbb{E}_t \sum_{j=0} \beta_i^j \widetilde{\pi}_i(q_{i,t+j}) = \max_{g_{i,t}} \sum_{j=0} \beta_i^j \pi_i(q_{i,t+j}) + \underbrace{\mathbb{E}_t \sum_{j=0} \beta_i^j \zeta_{t+j}}_{=0}$$

Consequently, under Assumptions 1 and 3, the optimal R&D investment (firm-level growth) satisfies the following optimality condition:

$$\psi \frac{s_i(x_{i,t})}{x_{i,t}} = \sum_{j=1} \beta_i^j \frac{\epsilon_{\pi q,t+j}}{1+g_{i,t}} \pi_{t+j}, \tag{A13}$$

which is the same condition as (A6). Following the same steps as in Section A.3 delivers

$$\epsilon = \frac{s}{\pi} \frac{1}{\psi - 1}$$

B.3 Financial Frictions

In this Appendix, we extend our baseline framework to allow for financial (or other) frictions, e.g. along the lines of Ottonello and Winberry (2025). We do so by considering a "wedge", $\lambda_{i,t} \geq 1$, which enters the condition for optimal innovation decisions. In this setting, equation (10) becomes:

$$\psi \frac{s_i(g_{i,t})}{g_{i,t}} = \sum_{j=1} \beta_i^j \lambda_{i,t+j} \frac{\epsilon_{\pi q,t+j}}{1+g_{i,t}} \pi_{t+j}. \tag{A14}$$

In the terminology of Ottonello and Winberry (2025), $\lambda_{i,t+j}$ represents the shadow value of funds. For unconstrained firms, $\lambda_{i,t} = 1$ and our baseline framework applies. For constrained firms, $\lambda_{i,t} > 1$ and firms benefit from innovation not only because growth increases future profits, but also because it loosens financing constraints. In particular, $\frac{\partial \lambda_{i,t}}{\partial q_{i,t}} \leq 0$. For the same reason, cheaper R&D brought about by more generous subsidies also leads to a loosening of financing constraints, $\epsilon_{i,\lambda,\tau} = \frac{\partial \lambda_{i,t}}{\partial \tau} \frac{\tau}{\lambda_{i,t}} \leq 0$.

Next, following the same steps as in A.3 and assuming that $\epsilon_{i,\lambda,\tau} \leq 0$ is constant over time, though possibly heterogeneous across firms, we can derive firms' micro-elasticities as:

$$
\begin{aligned}
\frac{\psi s(x_t)}{x_t}(\psi-1)\,\epsilon\frac{d\tau}{\tau} &= \sum_{j=1}\beta^j\left(\lambda_{t+j}\frac{\epsilon_{\pi q,t+j}}{1+g_t}\frac{\partial\pi_{t+j}}{\partial\tau}+\pi_{t+j}\frac{\epsilon_{\pi q,t+j}}{1+g_t}\frac{\partial\lambda_{t+j}}{\partial\tau}\right)d\tau\\
\frac{\psi s(x_t)}{x_t}(\psi-1)\,\epsilon\frac{d\tau}{\tau} &= \sum_{j=1}\beta^j\frac{\lambda_{t+j}\epsilon_{\pi q,t+j}\pi_{t+j}}{1+g_t}\left(\frac{s_{t+j}}{\pi_{t+j}}+\epsilon_{\lambda,\tau,t+j}\right)\frac{d\tau}{\tau}\\
\underbrace{\frac{\psi s(x_t)}{x_t}}_{MC}(\psi-1)\,\epsilon\frac{d\tau}{\tau} &= \left(\frac{s}{\pi}+\epsilon_{\lambda,\tau}\right)\underbrace{\sum_{j=1}\beta^j\frac{\lambda_{t+j}\epsilon_{\pi q,t+j}\pi_{t+j}}{1+g_t}}_{MB}\frac{d\tau}{\tau}\\
\epsilon_i &= \frac{1}{\psi-1}\left(\frac{s_i(x_i)}{\pi_i}+\epsilon_{i,\lambda,\tau}\right).
\end{aligned}
$$

C Further Details on Data

In this Appendix, we provide further details on the nature of the three datasets we use in our analysis: Compustat, BLADE and Orbis.

C.1 Compustat and Patent Information

As discussed in detail in Kogan et al. (2017), the inventor of a patented innovation can assign the granted property rights to another legal entity, for example a corporation. Therefore, granted patents may have, in addition to the inventor, an assignee, that is, one or more corporations or persons. Kogan et al. (2017) match corporate assignees of all U.S. patents to publicly traded U.S. corporations whose stock market returns can be found in the CRSP database. Finally, using the CRSP identifiers, we match the corresponding balance sheet information in the Compustat dataset. The resulting information allows us the calculate the number of patents for each firm in Compustat.

Following Bloom et al. (2013); Jaffe (1986), we use Cooperative Patent Classification (CPC) codes to measure similarity between firms in the technology space. We classify each patent using 3-digit CPC codes into one of 130 technology classes. Next, for each firm we compute the share of patents in all technology classes and compute the un-centered correlation between firm i's and firm j's patent shares denoted by $\alpha_{i,j} = \frac{T_i T_j'}{\sqrt{T_i T_i'}\sqrt{T_j T_j'}}$, where T_i is a 130 element vector in which each element corresponds to the number of patents granted to firm i in a given CPC class. Building on Bloom et al. (2013), we account for the firm's importance in the patenting network by weighting each observation by the total citations accrued to the given firm. Let c_j denote all citations attributed to patents assigned to a given firm-window cell. We measure spillovers benefiting a firm i as the proximity-weighted, citation-adjusted R&D expenses of all other firms, $\sigma_i = \sum_j \widetilde{\alpha}_{i,j}\frac{c_j}{C}\frac{s(x_j)^{\frac{1}{\psi}}}{S_i^{ext}}$, where $C = \sum_j c_j$ marks the total citation count.

C.2 BLADE

In what follows, we provide a brief description of the Business Longitudinal Analysis Data Environment (BLADE).

ABS Data Disclaimer. The results of these studies are based, in part, on data supplied to the ABS under the Taxation Administration Act 1953, A New Tax System (Australian Business Number) Act 1999, Australian Border Force Act 2015, Social Security (Administration) Act 1999, A New Tax System (Family Assistance) (Administration) Act 1999, Paid Parental Leave Act 2010 and/or the Student Assistance Act 1973. Such data may only used for the purpose of administering the Census and Statistics Act 1905 or performance of functions of the ABS as set out in section 6 of the Australian Bureau of Statistics Act 1975. No individual information collected under the Census and Statistics Act 1905 is provided back to custodians for administrative or regulatory purposes. Any discussion of data limitations or weaknesses is in the context of using the data for statistical purposes and is not related to the ability of the data to support the Australian Taxation Office, Australian Business Register, Department of Social Services and/or Department of Home Affairs' core operational requirements.

Legislative requirements to ensure privacy and secrecy of these data have been followed. For access to PLIDA and/or BLADE data under Section 16A of the ABS Act 1975 or enabled by section 15 of the Census and Statistics (Information Release and Access) Determination 2018, source data are de-identified and so data about specific individuals has not been viewed in conducting this analysis. In accordance with the Census and Statistics Act 1905, results have been treated where necessary to ensure that they are not likely to enable identification of a particular person or organisation.

Firm-level information. We use administrative firm tax records provided by the Australian Bureau of Statistics as part of the Business Longitudinal Analysis Data Environment (BLADE). BLADE covers the universe of businesses registered for the Goods and Services Tax with total sales exceeding AUD75,000, but does not include sole traders or partnerships that submit personal income tax returns instead of business income tax returns. The subset of BLADE used in this project sources data from the Australian Tax Office for financial years 2010 (July 1 2009-June 30 2010), 2011 and 2012.

Firm-level R&D is reported in the Business Income Tax (BIT) dataset, and is an accounting measure of expenditure subject to the R&D subsidy. Total sales (turnover) are reported in the Business Activity Statement. We define profits as operating profits, as reported in the BIT. The Longitudinal Indicative data items are used to categorize firms by industry codes, based on the 2006 Australian and New Zealand Standard Industrial Classification (ANZSIC).

C.3 Orbis

Orbis is a dataset managed by Bureau van Dijk that collects balance sheet information on both private and public companies across all industrialized countries. As of 2022, the Orbis dataset contains information on around 400 million companies and entities from more than 200 countries and territories, of which more than 99% are private companies. To access the Orbis dataset, we use WRDS services which allows us to retrieve up to 10 years of history of company information.

To retrieve the firm-level data, we filter the database by restricting attention to firm-year observations with positive R&D expenses and to companies registered in one of the 11 countries in the Appelt et al. (2025) dataset.[1] We focus on consolidated balance sheets (Orbis codes "C1", "C2", or "C3"). In calculating the average theoretical elasticity $\epsilon_j = \frac{s_j}{\pi_j}$ we follow the same steps as in the case of the Compustat data. Namely, we define profits π_j as sales net of cost of goods sold and R&D expenses. We restrict attention to firms with positive profits, positive revenues and positive cost of goods sold. To account for outliers, we drop observations with $\epsilon_j > 10$. The final sample consists of 16,713 firm-year observations.

D Further Details on Empirical Validation

In this Appendix, we provide further details and additional results on parts of our empirical validation in the main text.

D.1 Heterogeneity in R&D cost elasticities

As mentioned in the main text, we gauge the extent of potential heterogeneity in R&D cost elasticities across industries by estimating regression (17) separately for each 2 digit SIC sector. We restrict attention to sector-decade cells with at least 10 observations with positive R&D.[2] Figure A1 documents that we cannot reject $\psi = 2$, a common value used in the literature (see, e.g., Acemoglu et al. (2018)), for 20 out of 26 SIC sectors (77%).

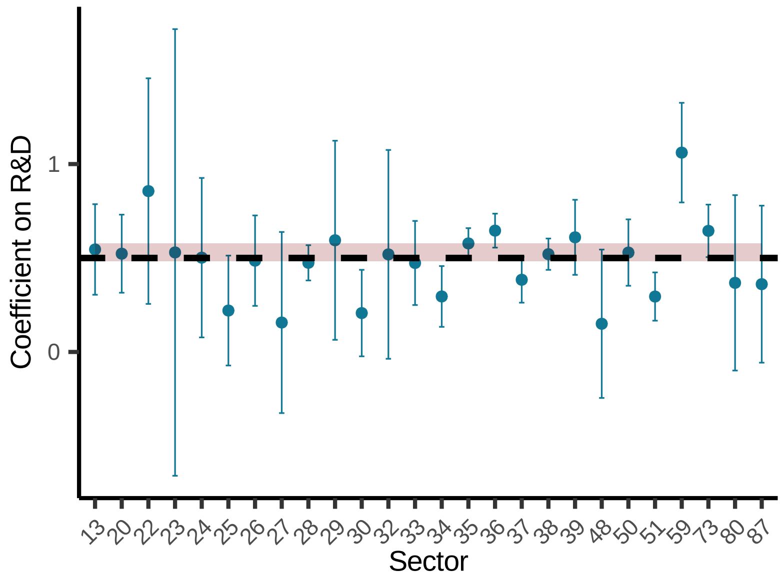
D.2 Further details on Assumption 2

Figure A2 presents the average change $\Delta_k \frac{s_{j,t}}{\pi_{j,t}} = \frac{s_{j,t+k}}{\pi_{j,t+k}} - \frac{s_{j,t}}{\pi_{j,t}}$ for various horizons k. The difference is statistically insignificant at most horizons and in all cases, quantitatively not meaningful. Panel (a) presents the results for data averaged over 5-year non-overlapping windows and Panel (b) the results for annual data.

[1] These countries are Australia, Belgium, Czechia, France, Italy, Netherlands, New Zealand, Norway, Portugal, Slovakia, Sweden.

[2] This requirement removes 86 sector-decade cells. The results of the full sample estimation deliver similar results with a few outlying sectors in terms of point estimates and precision.

Figure A1: Cost elasticity of R&D



The figure presents coefficients of regression 17 individually estimated in each 2-digit SIC sector (blue dots) and 95% confidence intervals (blue vertical lines). Red shaded area corresponds to 95% confidence interval of unconditional estimate of β using data pooled over all industries. Black dashed vertical line corresponds to $\beta = 0.5$ or, equivalently, $\psi = 2$.

Table A1 presents the estimates in regression (18) using annual data (corresponding to Table 2 in the main text that presents the results based on averaged data). The results obtained in annual data provide even stronger support to the assumption of constant s/π ratio than the results in averaged data.

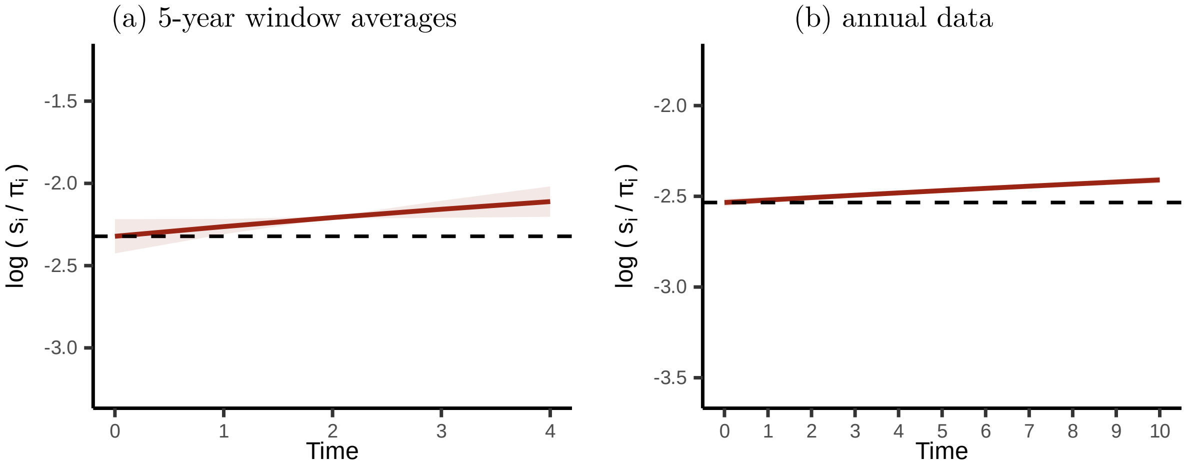
D.3 Further Details on BLADE Estimation

In what follows, we provide further details on the main specification of our BLADE estimation in the main text. In later subsections, we also provide additional robustness checks.

Institutional background. Prior to the 2012 R&D Tax Incentive reform, Australian firms were able to deduct 125% of R&D expenditure from taxable income, as well as an additional 50% of the portion of expenditure exceeding average expenditure over the previous three years.[3] Throughout the sample period, the corporate tax rate was 30% implying an effective R&D subsidy of $1.25 \times 0.3 = 0.375$. Firms with sales less than AUD5 million could instead choose to claim 30% of R&D expenditure as a refundable tax offset i.e. the firm was entitled to a cash refund of any unused offset amount if the

[3]We assume that there was no systematic difference between treatment and control groups with respect to the additional deduction in 2011 i.e. relative to average R&D expenditure over the previous three years, 2011 R&D expenditure was not systematically different between AUD 5-20 million firms and AUD 20+ million firms. The placebo tests in Table A2 provide support for this assumption.

Figure A2: R&D-to-profit ratio varies little over the firm lifecycle.



The figure presents the predicted level of $\log(s_i/\pi_i)$ based on estimated regression (18) with firm fixed effects. In Panel a), the horizontal axis corresponds to 5-year windows over which we averaged all firm-level outcomes. In Panel b), the horizontal axis corresponds to years. The solid line corresponds the projected level of the outcome variable. The shaded area marks the confidence interval. The projections are calculated for a hypothetical firm that is created in $t = 0$ and lives for T periods ($T = 5$ in averaged and $T = 11$ in annual data). The projections are normalized such that at time zero, the predicted value is equal to the mean outcome in the sample. The range of the vertical axis corresponds to the top and bottom quartiles of firm fixed effects in regression (18). The dashed horizontal line marks the sample mean of the outcome variable.

Table A1: Within-firm variation in R&D-to-profits ratio.

	R&D-to-profits		
time	0.007	0.004	-0.007
	(0.002)	(0.002)	(0.001)
time^2	-0.0001	-0.0001	0.0001
	(0.0001)	(0.0001)	(0.0001)
firm fixed effects	✓		
cohort fixed effects		✓	
sector fixed effects		✓	✓
Observations	66,114	61,905	61,905
R^2	0.75	0.38	0.35
Within R^2	0.002	0.0010	0.004

Note: The dependent variable is $\log\left(\frac{s_{j,t}}{\pi_{j,t}}\right)$ which implies that we only consider firms with positive R&D expenses and positive profits. Index t corresponds to a year and, therefore, the "time" variable corresponds to normalized firm age.

firm's tax liability was reduced to zero.

Following the policy change in 2012, firms with sales less than AUD20 million were eligible for a refundable 45% tax offset. By contrast, firms with sales exceeding AUD20 million were entitled to a non-refundable 40% tax offset.

Firms with sales between AUD5 million and AUD20 million therefore experienced a 20% increase in the effective R&D subsidy rate (changing from 37.5% to 45%). In our baseline difference-in-difference setup, we compare R&D expenditure among these businesses to firms with sales exceeding AUD20 million. For this latter group, the effective R&D tax subsidy increased by only 6.7% (from 37.5% to 40%).

Sample selection. In keeping with the Compustat analysis, we consider firms that report positive R&D expenditure and positive profits in both 2011 and 2012. To deal with outliers, we drop the top 5% of firms with regard to their R&D-to-profit ratio.

Further details on estimation methodology. For the purposes of the difference-in-difference estimation, we only consider firms with sales exceeding AUD5 million.[4] In addition, we only consider firms that remain in the same sales category (i.e. AUD5-20 million or AUD20+ million) across both 2011 and 2012 (the vast majority of firms). This approach ensures that the treatment and control groups are stable over time, and that the parallel trends assumption is not violated.

To convert the point estimate of β_1 from (19) to an R&D expenditure elasticity, we divide by 0.133, which represents the percentage difference in policy rate increases between the treated and control groups: $0.45/0.375 - 0.4/0.375 = 0.133$.[5]

D.4 Further Results in BLADE: Placebo Tests

The key identifying assumption in the difference-in-difference setup is the parallel trends assumption: in the absence of the policy reform, the R&D expenditure of AUD5-20 million sales firms and AUD20+ million sales firms would have changed by the same amount.

To test this, we consider a placebo treatment. In particular, we estimate regression (20) for 2010 and 2011 i.e. prior to the policy change in 2012. The sample is chosen in the same fashion as the baseline difference-in-difference of Table 4. Table A2 reports the results from the placebo regressions. Reassuringly, prior to the policy reform, the coefficient of interest is not significantly different from zero.

[4]Depending on firm profitability and R&D expenditure, firms with sales less than AUD5 million in 2011 may have been better off claiming the refundable tax offset (30%) rather than the non-refundable tax deduction (37.5%) or vice versa, complicating the calculation of firm-level responsiveness to the subsequent subsidy change.

[5]The elasticity calculation assumes that firms were not eligible for the additional 50% concession in 2011 i.e. at the firm level, 2011 R&D expenditure did not exceed the average of the previous three years. Taking the other extreme, we could instead assume that firms fully benefited from the additional 50%

Table A2: Elasticity of R&D expenditures: Placebo Tests

	(I)	(II)
AUD5-20 mil. sales $\times$ 2011, β_1	-0.009	-0.012
	(0.037)	(0.037)
Control for sales		✓
Observations	3472	3472

Notes: The table presents estimates from regression (19) in the period prior to the policy reform. The regression sample includes firms in years 2010 and 2011 with sales exceeding AUD5 million, those that report positive R&D expenditure, positive operating profits and remain in the same sales category across both 2010 and 2011. Standard errors in brackets are clustered at the firm level.

D.5 Further Results in BLADE: Fuzzy Difference-in-Differences

Our baseline difference-in-differences regression (19) estimates the intention to treat (ITT) i.e. the responsiveness of firms to the policy reform based on subsidy *eligibility*. This is irrespective of whether the firm actually took up the more generous subsidy. In this section, we consider a sub-sample of firms for which we *observe the choice* of subsidy.

We find that some firms in the AUD5-20 million category did not choose the more generous 45% offset, instead opting for the 40% offset.[6] We, therefore, employ a 'fuzzy' DiD design (the fuzziness arising from the fact that some units in the treatment group do not take the treatment), using the Wald-DiD estimator.

In addition to the standard parallel trends assumption, de Chaisemartin and D'Haultfoeuille (2018) highlight further assumptions required for the Wald-DiD to estimate the local average treatment effect (LATE). For our two-period setting, in which all units are untreated in the pre-period (i.e. 2011), we require that either (1) there is a stable percentage of treated units in the control group, or (2) the LATE is homogeneous across both treatment and control groups. Since the control group is ineligible for the 45% offset, assumption (1) is satisfied.

As such, we run an instrumental variable regression of R&D expenditure on the 45% offset, with year and sales category as included instruments, and the interaction of the two as the excluded instrument. In particular, the first stage is:

$$\mathbb{1}_{45\%} = \kappa_0 + \kappa_1 \mathbb{1}_{\$5-20M} \times \mathbb{1}_{2012} + \kappa_2 \mathbb{1}_{\$5-20M} + \kappa_3 \mathbb{1}_{2012} + \kappa_4 X_{i,t} + \nu_{i,t}, \tag{A15}$$

where $\mathbb{1}_{45\%}$ is an indicator function equal to one if firm i took the 45% offset and all other

concession in 2011, in which case we would divide by $0.45/0.525 - 0.4/0.525 = 0.095$.

[6] Survey evidence suggests that some firms remain unaware of R&D subsidies for many years after such policies are introduced (Thomson and Webster, 2012).

Table A3: Fuzzy Differences-in-Differences

	(I)	(II)
Panel A: First Stage		
	Dependent Variable: 45% Offset	
AUD5-20 mil. sales × 2012, κ_1	0.505	0.505
	(0.024)	(0.024)
F-statistic	441.699	441.796
Panel B: Second Stage		
	Dependent Variable: log(R&D)	
45% Offset, θ_1	0.317	0.294
	(0.090)	(0.088)
Control for sales		✓
Observations	2362	2362

Notes: The table presents estimates from an instrumental variable regression of log R&D expenditure on the 45% offset, with year and sales category as included instruments, and the interaction of the two as the excluded instrument. The regression sample includes firms in years 2011 and 2012 with sales exceeding AUD5 million, those that report positive R&D expenditure, positive operating profits, remain in the same sales category across both 2011 and 2012 and for which the choice of subsidy is observed. Standard errors in brackets are clustered at the firm level.

variables are the same as in the main text.

From regression (A15), we recover the predicted value, $\hat{\mathbb{1}}_{45\%}$, and estimate the second stage:

$$\log(R\&D)_{i,t} = \theta_0 + \theta_1 \hat{\mathbb{1}}_{45\%} + \theta_2 \mathbb{1}_{\$5-20M} + \theta_3 \mathbb{1}_{2012} + \theta_4 X_{i,t} + u_{i,t}, \tag{A16}$$

where the coefficient θ_1 is the LATE among compliers i.e. AUD5-20 million firms that took the 45% offset.

Table A3 presents the results from the Wald-DiD. First, note that the estimates of θ_1 (the LATE) are greater than those of β_1 (the ITT) in Table 4. This reflects the fact that not all firms took advantage of the policy. Next, focusing on our preferred specification (column II) and using the estimated standard errors to compute an upper and lower bound, the results suggest that the R&D expenditure elasticity lies between $\epsilon_s \in (1.55, 2.87)$ with a point estimate of 2.21.[7] The average R&D-to-profit ratio among complier AUD5-20 million firms is 0.992, implying an R&D expenditure elasticity of $\epsilon_s = 1.98$ (using $\psi = 2$). This falls within the estimated bounds.

[7] To convert the point estimate of θ to an R&D expenditure elasticity, we divide by 0.133 which represents the percentage difference in policy rate increases between the 45% and 40% offset groups (0.45/0.375 relative to 0.4/0.375). We obtain the lower and upper bound as $(0.294 \pm 0.088)/0.133$.

E Further Details on Theoretical Validation

The previous appendix discussed validation of our key assumptions using three different firm-level micro-data sets. To give further justification to the theoretical approach, we now examine the key assumptions through the lens of a quantitative model of endogenous growth in which Assumption 2 is not satisfied. We build on the model developed in Ignaszak and Sedláček (2025) that features imperfect scaling of firm value with respect profits. As a result, firms in the model exhibit lifecycle dynamics and deviate from Gibrat's law. The deviations are particularly strong among young and small firms. The model parameters - including the degree of deviation from perfect scaling - are estimated using the Compustat data.

In what follows, we simulate the model and compute the model-predicted micro-elasticities. We then compare those to our theoretical ones (based on Assumption 2, which does not hold in the model). Crucially, we show that the theoretical micro-elasticities remain to provide a good approximation to the model-predicted ones.

E.1 Brief Model Summary and Methodology Description

In the model, firms invest in R&D in order to increase their productivity as in the quality ladder model in the spirit of Grossman and Helpman (1991). Furthermore, in order to generate endogenous deviation from Gibrat's law, the model features frictional customer base accumulation. Firms can attract new customers by direct advertising spending or indirectly through lower prices. For further details, please refer to Ignaszak and Sedláček (2025).

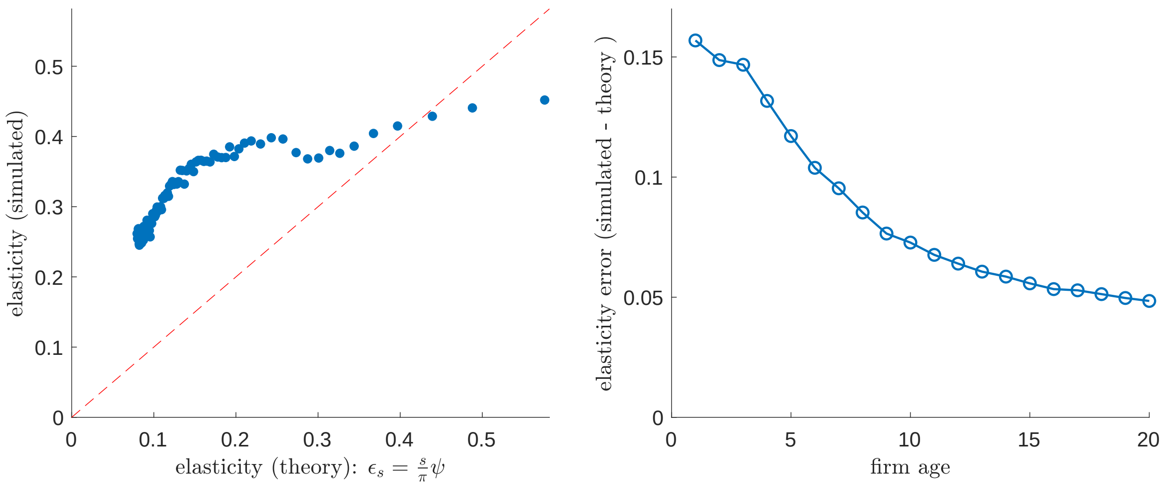
In the model, we simulate a one-time, permanent, and unanticipated change to the R&D subsidy. The subsidy is funded by a non-distortionary, lump sum tax on the households. We consider partial equilibrium responses, consistent with the theory developed in Section 2. In the context of Ignaszak and Sedláček (2025), this means that we keep aggregate wage, aggregate spending, and aggregate growth fixed at the pre-reform level.

When calculating the response of the R&D expenses at the firm level, we consider two cases: (i) we freeze the firm decisions related to customer accumulation at the pre-reform levels and (ii) we allow for full adjustment along all investment margins. We show that in both scenarios (i) and (ii), our sufficient statistics approach is well validated.

To gauge the accuracy of our sufficient statistics approach, we compare quantitatively the model implied elasticity, $\frac{s(x,\tau')-s(x,\tau)}{s(x,\tau)}\frac{\tau}{\tau'-\tau}$ to the theoretical sufficient statistic $\epsilon_{i,s} = \psi\frac{s_{i,t}(\tau)}{\pi_{i,t}(\tau)}$, where τ is the baseline level of R&D subsidy and $\tau' = 1.1 \times \tau$ is the new level.[8]

[8] The subsidy grows by 0.2 percentage point, from 5.6% in the baseline to 5.8%. We simulate a somewhat large relative increase in the subsidy to make sure that the absolute change is large enough so that any numerical inaccuracies in the model solution dwarf the resulting elasticities of R&D expenses. The baseline level of subsidy is chosen to match 0.12% of GDP spent on R&D subsidies in the US. See

Figure A3: Comparison of model-implied R&D spending elasticity with the sufficient statistic.



Notes: The figure presents the results simulating R&D subsidy in the model in Ignaszak and Sedláček (2025) with customer base adjustment. Panel (a) plots the theoretical elasticity $\epsilon_s = \psi\frac{s}{\pi}$ against the model simulated one, $\frac{s(x,\tau')-s(x,\tau)}{s(x,\tau)}\frac{\tau}{\tau'-\tau}$. Dashed red line corresponds to 45 degrees. The correlation between the two series is 0.65. In Panel (b) we show the difference between the simulated and the theoretical elasticities as a function of firm age. Positive values indicate that the statistics approach under-predicts the simulated elasticities. We show the results in the scenario in which firms adjust customer accumulation investment.

Towards this end, we simulate a cross section of firms starting from the stationary distribution in the equilibrium under the baseline subsidy level τ. Then, we solve for new policy functions for the R&D investment and customer base accumulation in partial equilibrium, that is, given the baseline level of prices and aggregate income level and aggregate income growth. To account for the long-run impact of the subsidy change, we solve for the new firm value function consistent with the new subsidy level. To be consistent with the empirical approach in Compustat, we define profits as revenues net of variable (labor) cost and R&D expenses.[9] Our statistics are based on a sample of 10000 firms drawn from the stationary distribution. The final sample used for calculation of micro-elasticities consists of 8910 firms which decided to continue operating after the subsidy introduction.[10]

Ignaszak and Sedláček (2025) for more details.

[9]As in Compustat, we drop observations with R&D/sales above 1000%, revenue growth rate above 1000%, theoretical and simulated elasticities above 10 as well as firms with negative profits or negative growth rates. To make sure that our results are not driven by extreme observations, potentially arising due to numerical inaccuracy, we trim top and bottom 1% of observations in terms of profits, R&D expenses, revenue, as well as simulated and theoretical elasticites.

[10]The model features endogenous exit induced by fixed operating costs.

E.2 Model Validation Results

Figure A3 presents the results. In Panel (a), we plot the the average theoretical elasticity (sufficient statistic), $\epsilon_s = \psi \frac{s}{\pi}$, and the simulated responses of firms (model-predicted elasticities), $\frac{s(x,\tau')-s(x,\tau)}{s(x,\tau)} \frac{\tau}{\tau'-\tau}$. Observations correspond to percentiles of the distribution of ϵ_s and the corresponding mean simulated elasticities among firms belonging to the particular percentile of the theoretical distribution. The correlation between our sufficient statistic approach and the simulated elasticities is very high: equal to 0.62 in scenario (i) (no customer base adjustment) and 0.65 in scenario (ii) (with endogenous customer base adjustment, scenario visible in Figure A3). Quantitatively, the implied elasticities are well aligned with those estimated using Compustat data in Section 5.

In Panel (b), we can see that the sufficient statistic approach gets more accurate for older firms. This is in line with the intuition that the concavity of the firm value function is particularly strong for young, fast growing firms. Note that the simulated elasticity tends to be *larger*, for younger firms. This means that our approach tends to *underestimates* the benefits of supporting young firms. Overall, however, despite its simplicity, our sufficient statistic approach presents a good description of the simulated elasticities implied by the model of Ignaszak and Sedláček (2025) in which Assumption 2 does not hold.

F Further Empirical Results and Robustness

In this Appendix, we provide additional results and robustness checks for some of our key findings.

F.1 Further Results: Small vs Young Firms

Table A4 documents that the relatively good performance of small firms in Table 7 is purely driven by small-young firms. For completeness, we repeat the remaining entries in the original table.

F.2 Further Results: Standard Errors

In this section we quantify the statistical significance of the estimated Bangs for the Buck and the difference in relative cost-effectiveness of the targeted subsidies between groups of firms. Let the population value of the Bang and the Buck be denoted by $\overline{B}_k = \sum_{j \in \overline{\Omega}_k} b_j$ and $\overline{C}_k = \sum_{j \in \overline{\Omega}_k} c_j$, respectively, where $\overline{\Omega}_k$ marks the theoretical population of firms of type k from which the Compustat data is sampled. Assume that firm level components

Table A4: Decomposition of the Relative Bang for the Buck within small firms.

Firm group	Relative Bang & Buck			Bang Components				Buck Components		
	A_k/A	B_k/B	C_k/C	m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.94	0.77	0.82	0.35	0.12	0.06	4.35	0.77	0.36	1.01
Small	1.39	0.09	0.06	0.03	0.08	0.10	6.50	0.05	0.26	2.03
Young	2.09	0.13	0.06	0.05	0.17	0.11	1.94	0.05	0.26	1.83
Small & Young	2.37	0.04	0.02	0.01	0.20	0.27	1.91	0.01	0.30	1.76
Small & Old	1.02	0.05	0.05	0.02	0.03	0.05	23.14	0.04	0.25	2.16
Gazelles	2.68	0.47	0.17	0.08	0.31	0.40	0.76	0.14	0.31	1.61
All	1.00	1.00	1.00	1.00	0.06	0.04	6.69	1.00	0.21	1.50

Note: The table reproduces Table 7 with addition of two groups of firms small-young and small-old. The former is defined as all firms below median sales and less than 6 years since IPO. The latter groups captures all remaining small firms. See also notes to Table 7.

are normally distributed. Then, the joint distribution converges in distribution to

$$\frac{1}{N_k}\begin{pmatrix} B_k - \overline{B}_k \\ C_k - \overline{C}_k \end{pmatrix} \to N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{B,k}^2 & \sigma_{B,C,k} \\ \sigma_{B,C,k} & \sigma_{C,k}^2 \end{pmatrix}\right)$$

Consider now the ratio $\frac{B_k}{C_k} = \frac{B_k^* + \overline{B}_k}{C_k^* + \overline{C}_k}$ where B_k^* and C_k^* are zero mean Gaussian random variables.

Applying the delta method, we obtain the expression for the asymptotic variance of the ratio $A_k = \frac{B_k}{C_k}$

$$\text{var}(A_k) = \frac{1}{\overline{C}_k^2}\left(\sigma_{B,k}^2 + \frac{\overline{B}_k^2}{\overline{C}_k^2}\sigma_{C,k}^2 - 2\frac{\overline{B}_k}{\overline{C}_k}\sigma_{B,C,k}\right). \tag{A17}$$

Table A5 presents standard errors of the Bangs for the Buck of the form $\pm\alpha\sqrt{\widehat{\text{var}}(A_k)}$, where $\widehat{\text{var}}(A_k)$ is the consistent estimate of $\text{var}(A_k)$ defined in (A17), in which population moments are replaced by sample analogs and where α corresponds to a percentile of the normal distribution. The implied errors are very tight and all differences between groups are statistically significant.

However, the resulting confidence intervals may become distorted in finite samples. To see why, note that we can write $\frac{B_k}{C_k} = \frac{\overline{B}_k}{\overline{C}_k}\frac{1+B_k^*/\overline{B}_k}{1+C_k^*/\overline{C}_k}$. If the population value of the cost $\overline{C}_k \geq 0$ in some subgroup k is quantitatively close to zero and therefore highly skewed, the normal approximation is inaccurate. This is precisely the case in our application where individual values of cost are non-negative, and the sum may be arbitrarily close to zero. To address potential inaccuracy of the asymptotic first-order approximation (the delta method), below we provide bootstrap standard errors.

Table A5: Asymptotic approximation standard errors for the relative Bang for the Buck.

	A_k/A	Lower bound of 95% CI	Upper bound of 95% CI
R&D-int.	0.94	0.93	0.94
Small	1.39	1.39	1.39
Young	2.09	2.08	2.10
Gazelles	2.68	2.65	2.70
All	1.00	1.00	1.00

Notes: The table presents relative Bangs for the Buck and the bootstrap confidence intervals of the form $\pm 1.96\sqrt{\text{var}(A_k)}$ where $\text{var}(A_k)$ is defined in (A17).

Table A6: Bootstrap standard errors for the relative Bangs for the Buck.

	A_k/A	Lower bound of 95% CI	Upper bound of 95% CI
R&D-int.	0.94	0.77	1.12
Small	1.39	1.27	1.54
Young	2.09	1.70	2.61
Gazelles	2.68	2.32	3.12
All	1.00	0.86	1.16

Notes: The table presents relative Bangs for the Buck and the standard errors of the form $\left[q_{2.5}^{A_k}, q_{97.5}^{A_k}\right]$, where $q_x^{A_k}$ is the x-th percentile of the bootstrap distribution of the Bangs for the Buck A_k. We use 1000 bootstrap samples.

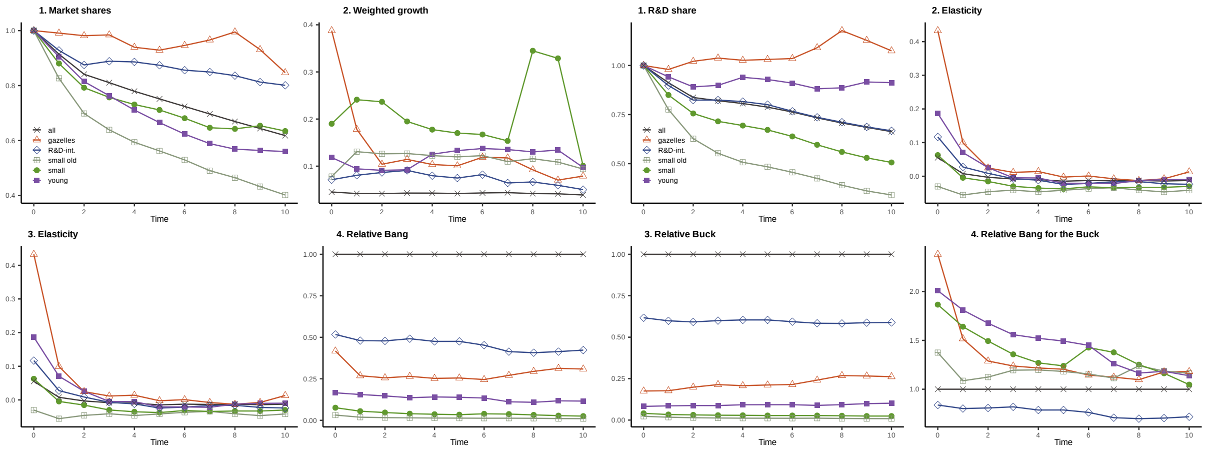
Bootstrap standard errors For each firm group, we draw with replacement observations from the same firm group to obtain a bootstrap sample of the same size as the original sample. In each of the 1000 bootstrap samples, we recompute the statistics of interest. In Table A6, we report the confidence interval based on the percentiles of the distribution of bootstrap Bangs for the Buck. While the confidence intervals are much wider than asymptotic standard errors, our main results remain statistically significant. Young firms are a statistically significantly more cost-effective target than small firms. Targeting gazelles is more than twice as cost-effective as the uniform subsidy. R&D intensive firms are a significantly worse target than all other groups of firms we consider in the application.

F.3 Further Results: Dynamics of the Bangs for the Buck

In this section we present more details pertaining to the dynamics of the estimated Bangs for the Buck across our firm groups. Our goal is to estimate how the relative Bang and Buck changes over time for fixed firm groups. In other words, is supporting firms that were gazelles one year or two years ago as sound a policy as supporting the current gazelles? To answer this question, we pool all firms and classify them into groups in the

Figure A4: Persistence of the Bang for the Buck and its components using unbalanced panel

(a) Drivers of the Bang (b) Drivers of the Buck



Note: Figure presents components of the Bang and the Buck using annual data and an unbalanced panel of firms. Sub-panel D. in panel (b) reproduces Panel (a) in Figure 3. See also notes to Figure 3.

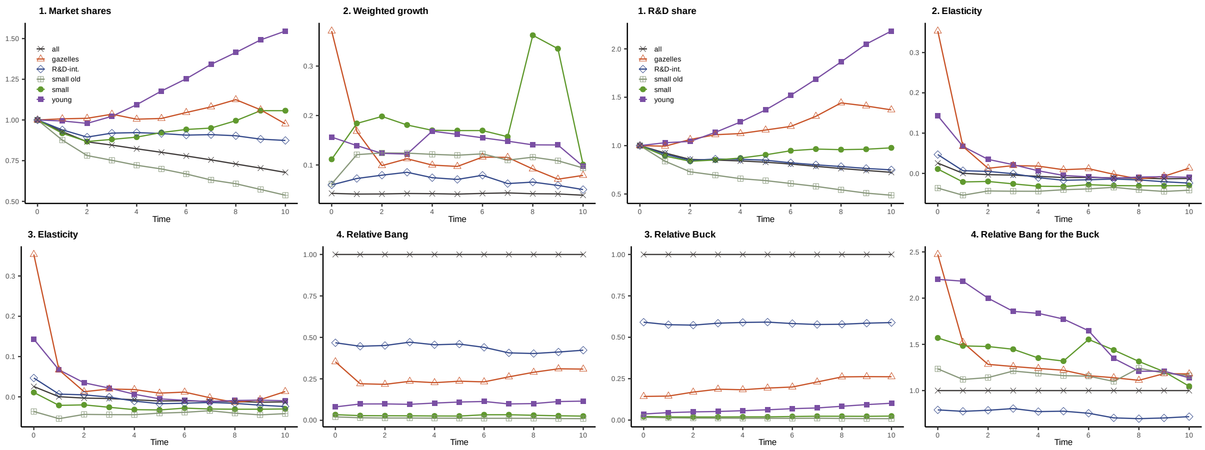
same manner as in Section 3.4. Then, keeping this classification fixed, we track firms over time and estimate all the components of the Buck and the Bang. For example, we estimate the efficiency of supporting firms that were gazelles one year ago, two years ago, etc. Or, in the case of window-averaged data, in the previous window, two windows ago, etc. As reported in the main text, young firms tend to exhibit persistently high benefits-to-cost ratio and stand out as excellent targets for R&D subsidies. Gazelles on the other hand, while the most cost-effective target from an instantaneous perspective, exhibit a rapid drop over time in their relative Bang for the Buck. To understand the forces behind the patterns in the Bangs for the Buck, in this section we present the evolution of the components of this aggregate measure.

Dynamics of Bang for the Buck Components. Figure A4 presents the evolution of the components of the Buck and the Buck over time in an unbalanced panel of firms. Panel (a) reports the components of the Bang. Sub-panel 1. shows the market share m across firm groups, normalized to 1 at time $t = 0$. The panel illustrates that small and young firms initially gain market share, but all firms converge to a similar size in the long run. Old firms tend to shrink on average, as illustrated by a sharp decline in the market share of all firms and small-old businesses. This result is consistent with evidence on the universe of firms in the US, reported in Haltiwanger et al. (2013): the net growth in the economy is entirely due to young firms.

Next in sub-panel 2., we can see that the size-weighted growth of gazelles drops quickly over time. In particular, firms that were classified as gazelles 2 years ago grow at a slower rate than those that were small or young two years ago. Small firms exhibit the highest

Figure A5: Persistence of the Bang for the Buck and its components using balanced panel

(a) Drivers of the Bang (b) Drivers of the Buck



Note: Figure presents components of the Bang and the Buck using annual data and a balanced panel of firms. Sub-panel D. in Panel (b) reproduces Panel (b) in Figure 3. See also notes to Figure 3.

growth rates from the long run perspective. However, this is driven entirely by the fact that most of them are young. This can be seen from the poor growth trajectory of small-old firms. Sub-panel 3. illustrates convergence in Bang-relevant elasticities. Note that after about the first 2 years, these elasticities remain relatively stable, consistent with Assumption 2. Finally, sub-panel 4. concludes by plotting the relative Bangs over time.

Next, Panel (b) on the right-hand side reports the components of the Buck and the total relative Bangs for the Buck in sub-panel 4. Note that sub-panel 4. replicates Panel (a) in Figure 3. Here, the most striking result is the stable R&D share among gazelles. Recall from our previous discussion that their the revenue growth and market shares are gradually declining, reducing the Bang. In combination with their stable R&D share, this is responsible for the decline in their relative Bang for the Buck visible in sub-panel 4.

Figure A5 presents the same set of results as above, but generated from the balanced panel of firms. That is, we restrict attention to the set of firms for which we can obtain all necessary balance sheet items in at least 10 consecutive years. Here sub-panel 4. in Panel (b) corresponds to the results reported in Figure 3 in the main text. By focusing on a balanced panel, we select firms based on their ex-post success. This is particularly visible when inspecting the time paths for young firms: ex-post successful young firms grow rapidly increasing their market and R&D shares, as can be seen in sub-panels 1. in Panels (a) and (b) in Figure A5. This elevates their relative Bang for the Buck throughout the horizon of interest. The dynamics of the remaining firm groups is broadly in line with the results in the unbalanced panel described above.

Table A7: Decomposition of the Bang for the Buck for employment-based growth

Firm group	Relative Bang & Buck A_k/A	B_k/B	C_k/C	Bang Components m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	Buck Components r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.95	0.68	0.72	0.23	0.12	0.08	4.92	0.65	0.63	0.70
Small	1.31	0.06	0.05	0.01	0.06	0.10	17.20	0.03	0.47	1.82
Young	1.71	0.17	0.10	0.07	0.14	0.07	4.07	0.07	0.45	1.45
Gazelles	2.28	0.47	0.20	0.08	0.32	0.31	0.71	0.15	0.50	1.25
All	1.00	1.00	1.00	1.00	0.05	0.02	12.58	1.00	0.33	1.05

The table reproduces Table 7 when using employment growth, rather than sales, as the measure of firm growth. See also the notes to Table 7.

F.4 Robustness: Employment Growth

Table A7 shows that if firm-level growth rate g_j is measured using employment, rather than revenues, the results remain quantitatively and qualitatively unchanged. Note that we have defined gazelles as firms with a high revenue growth, which does not automatically imply rapid employment growth. Nevertheless, gazelles report the highest average employment growth of the considered firm groups. Similarly, small firms remain defined as the firms with below median sales. All in all, these results further strengthen our main message that gazelles and young firms are the most cost-effective firm group to support.

F.5 Robustness: Operating Profits

Table A8 indicates that our results are robust to an alternative definition of profits. In the main text, we use a model-consistent measure defined as revenues, less costs of goods sold and R&D expenses. A broader profitability measure, operating income before depreciation and amortization (Compustat mnemonic `oibdp`) includes other expenses, such as marketing. When using this measure of profitability in the definition of elasticity ϵ, all results remain qualitatively and quantitatively unchanged.

F.6 Robustness: Definition of Aggregates

Table A9 reports the decomposition of the Bangs for the Buck when we use an alternative definition of the aggregate variables. For the Domar weights m we use US GDP (as opposed to the total sales of all firms in our data) and in the R&D r we use the aggregate R&D spending in the US (as opposed to the total R&D expenses of all firms in our data). Note that relative measures in the three left-most columns are unaffected by the choice of the denominator in the Domar weight and R&D share. The components of the Bang and Buck in the remaining columns are very similar to those reported in the main text. By ignoring firm heterogeneity, we severely underestimate the impact of R&D subsidies.

Table A8: Decomposition of the Bang for the Buck with alternative profit measure.

	Relative Bang & Buck			Bang Components				Buck Components		
Firm group	A_k/A	B_k/B	C_k/C	m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.92	0.76	0.83	0.37	0.05	0.05	14.97	0.78	0.33	1.05
Small	1.04	0.05	0.04	0.02	0.02	0.07	32.49	0.04	0.24	2.10
Young	1.58	0.07	0.04	0.04	0.08	0.10	4.48	0.04	0.25	1.57
Gazelles	2.34	0.36	0.15	0.12	0.22	0.35	0.69	0.14	0.22	1.75
All	1.00	1.00	1.00	1.00	0.03	0.04	17.81	1.00	0.19	1.53

The results in the table are based on the profits defined as the income before interest and depreciation. The sample of firms differs from the one underlying Table 7 due to missing profit data. See also the notes to Table 7.

Table A9: Decomposition of the Bang for the Buck with Alternative Aggregates

	Relative Bang & Buck			Bang Components				Buck Components		
Firm group	A_k/A	B_k/B	C_k/C	m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.94	0.77	0.82	0.11	0.14	0.06	3.84	0.59	0.38	0.98
Small	1.39	0.09	0.06	0.01	0.09	0.10	6.04	0.04	0.27	2.01
Young	2.09	0.13	0.06	0.02	0.19	0.10	1.89	0.03	0.28	1.78
Gazelles	2.68	0.47	0.17	0.02	0.34	0.40	0.71	0.11	0.34	1.51
All	1.00	1.00	1.00	0.33	0.07	0.04	5.92	0.76	0.22	1.47

The table reproduces Table 7 when we use nominal GDP as the denominator in the Domar weight and the aggregate US-wide R&D spending in the denominator of the R&D shares r. See also the notes to Table 7.

F.7 Robustness: Time Averaging

The goal of this section is to document that the results obtained in the main text are not an artifact of the averaging procedure that we employ. Table A10 presents the results using annual data rather than averaging over non-overlapping 5-year windows. Each firm-year cell is treated as an independent observation. The results in Table A10 are very similar to those in Table 7 in the main text.

F.8 Robustness: Non-negative Growth Rates

Table A11 documents that the relative ranking of targeted subsidies remains intact when we restrict attention to the firms with positive revenue growth rates. Note that the first column of Table A11 is the same as in the first column of the Table 8 where we report R&D spillovers. The reason is that we in both cases we use only firms with positive growth rates.

Table A10: Decomposition of the Bang for the Buck using annual data.

Firm group	Relative Bang & Buck A_k/A	B_k/B	C_k/C	Bang Components m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	Buck Components r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.87	0.58	0.67	0.21	0.09	0.08	7.99	0.60	0.60	0.73
Small	1.58	0.10	0.06	0.01	0.05	0.18	16.85	0.04	0.46	1.86
Young	2.01	0.22	0.11	0.07	0.16	0.12	3.28	0.09	0.44	1.31
Gazelles	2.56	0.44	0.17	0.12	0.42	0.39	0.45	0.15	0.42	1.17
All	1.00	1.00	1.00	1.00	0.05	0.04	10.43	1.00	0.32	1.05

The table reproduces Table 7 when we restrict attention to firms with positive growth rates. See also the notes to Table 7.

Table A11: Decomposition of the Bang for the Buck for $g_j > 0$

Firm group	Relative Bang & Buck A_k/A	B_k/B	C_k/C	Bang Components m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	Buck Components r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.99	0.81	0.81	0.33	0.37	0.13	1.01	0.76	0.37	1.05
Small	1.52	0.08	0.05	0.02	0.27	0.22	1.24	0.04	0.27	2.00
Young	2.27	0.15	0.07	0.06	0.28	0.21	0.88	0.06	0.28	1.55
Gazelles	2.40	0.61	0.25	0.13	0.31	0.40	0.76	0.21	0.31	1.61
All	1.00	1.00	1.00	1.00	0.21	0.10	0.91	1.00	0.21	1.53

The table reproduces Table 7 when we restrict attention to firms with positive growth rates. See also the notes to Table 7.

F.9 Robustness: Negative Profits

In the main text, we consider only firms with positive profits. The reason is that the environment used to derive the theoretical R&D elasticity ϵ implicitly assumes that all firms record positive profit. In this Appendix, we document that our results are robust to two ways of dealing with firms reporting negative profits. First, adjusting the definition of the micro-elasticity. Second, using firm value instead.

Including firms with negative profits. If firms report negative profits, their micro-elasticity would imply a reduction in R&D expenses following a lowering of the price of innovation. To deal with this, we impose that the micro-elasticity for firms with negative profits is given by $\epsilon = -\frac{s}{\pi} > 0$. In other words, we treat the R&D-to-profit ratio as informative about the R&D intensity of firms, but we make sure that the elasticity is always positive. Table A12 shows that our results hold in the extended sample.

Using firm value. As a second approach, we consider working with firm value directly. Let us begin by repeating the R&D optimality condition but where we purposefully work

Table A12: Decomposition of the Bang for the Buck including negative profits

	Relative Bang & Buck			Bang Components				Buck Components		
Firm group	A_k/A	B_k/B	C_k/C	m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.88	0.61	0.69	0.21	0.08	0.08	10.12	0.60	0.81	0.62
Small	1.34	0.13	0.10	0.01	0.05	0.19	23.25	0.05	0.65	1.89
Young	1.87	0.25	0.14	0.07	0.18	0.13	3.56	0.09	0.65	1.20
Gazelles	2.46	0.45	0.18	0.12	0.59	0.40	0.35	0.15	0.59	0.96
All	1.00	1.00	1.00	1.00	0.04	0.04	12.04	1.00	0.44	0.84

The table reproduces Table 7 when we include firms reporting negative profits. See also the notes to Table 7.

with firm values:

$$\psi \frac{s(x_t)}{x_t} = \frac{\partial v_t}{\partial x_t} = \epsilon_{v,x} \frac{v_t}{x_t}. \tag{A18}$$

The RHS of the above equation is the change in firm value brought about by investing more into R&D (i.e. growth), where $\epsilon_{v,x} = \frac{\partial v_t}{\partial x_t} \frac{x_t}{v_t}$ is the associated elasticity. Then, totally differentiating the above w.r.t. $d(1-\tau)$, noting that at the margin that is equivalent to $-d\tau$, we can write

$$\begin{aligned}
\overbrace{\psi \frac{s(x_t)}{x_t}}^{MC} \epsilon(\psi - 1) \frac{d(1-\tau)}{1-\tau} &= \overbrace{\frac{\epsilon_{v,x}}{x_t}}^{MB} v_t \frac{\partial v_t}{\partial(1-\tau)} \frac{1-\tau}{v_t} \frac{d(1-\tau)}{1-\tau} \\
\epsilon(\psi - 1) &= -\sum_j \beta^j \widetilde{s}_{t+j}(1-\tau) \frac{1}{v_t} \\
\epsilon &= \frac{1}{\psi - 1} \frac{\sum_j \beta^j s_{t+j}}{v_t}
\end{aligned} \tag{A19}$$

Notice that under the assumption that s_t/π_t is constant at the firm level, then the above boils down to our original micro-elasticity. This is because $\sum_j \beta^j s_{t+j} = \sum_j \beta^j \frac{s_{t+j}}{\pi_{t+j}} \pi_{t+j} = \frac{s}{\pi} \sum_j \beta^j \pi_{t+j} = \frac{s}{\pi} v$. In other words, the micro-elasticity is the ratio of the net present value of all future R&D expenditures, relative to firm value.

We measure v_t as the company-level consolidated market value.[11] As for the present discounted value of future R&D expenses, we assume that the firm expects them to remain constant over time, $\sum_j \beta^j s_{t+j} = \frac{s_t}{1-\beta}$.[12] We set the discount factor to $\beta = 0.96$. Table A13 shows that using this alternative measure does not materially change our results.

[11] We use Compustat variable `mkvalt` which captures sum of all issue-level market values, including trading and non-trading issues. We impute missing values with the product of fiscal year closing price of the public stock, `prcc_f`, and the number of common shares outstanding, `csho`.

[12] Recall that the time period refers here to an average value within a 5-year window.

Table A13: Decomposition of the Bang for the Buck including negative profits, alternative elasticity measure

	Relative Bang & Buck			Bang Components				Buck Components		
Firm group	A_k/A	B_k/B	C_k/C	m_k	$\bar{\epsilon}_k^B$	g_k	θ_k^g	r_k	$\bar{\epsilon}_k$	θ_k^s
R&D-int.	0.86	0.54	0.62	0.19	0.14	0.07	5.42	0.55	0.62	0.69
Small	1.96	0.07	0.04	0.01	0.08	0.18	11.18	0.02	0.44	1.95
Young	2.23	0.15	0.07	0.04	0.19	0.13	3.00	0.05	0.44	1.29
Gazelles	2.54	0.46	0.18	0.12	0.41	0.38	0.46	0.15	0.41	1.17
All	1.00	1.00	1.00	1.00	0.07	0.05	6.27	1.00	0.31	1.02

The table reproduces Table 7 when we include firms reporting negative profits. To calculate the elasticity, we use formula (A19). In the formula, we let $\sum_j \beta^j s_{t+j} = \frac{s_t}{1-\beta}$ where $\beta = 0.96$. The firm value v is measured as market value. See also the notes to Table 7.

F.10 Robustness: Spillovers with patent value-adjusted R&D expenses

To quantify technological spillovers between firms, we assume that the extent of spillovers generated by a firm j on any firm i depends on how close to each other these firms are in the technology space (measured by proximity measure $\alpha_{i,j}$ defined in the main text). Moreover, to accurately gauge the spillovers, we adjust R&D expenses for the quality of innovation a given firm generates. Intuitively, for a given amount of resources devoted to R&D, the more technological spillovers generated by a given firm, the more groundbreaking is the type of innovation in which the firm is engaged. In the main text, we use citation counts to quantify the quality of R&D expenses in a given firm. In this appendix, we show that the results remain quantitatively unchanged when we use patent value estimated in Kogan et al. (2017) as the measure of R&D quality.

Table A14 reports the results. Gazelles remain dominant in terms of their relative Bang for the Buck. The ranking of firm groups is the same as the baseline specification in which we used citation counts to measure R&D quality.

Table A14: Relative Bang for the Buck including spillovers with patent value as the measure of quality

Firm group	$\frac{A_k}{A}$	$\frac{A_k^{spill}}{A^{spill}}$	$\frac{B_k^{int}}{B_k}$	$\frac{B_k^{ext}}{B_k}$	$\frac{A_k^{spill}}{A^{spill}}$	$\frac{B_k^{int}}{B_k}$	$\frac{B_k^{ext}}{B_k}$
		$\omega = 0.85$			$\omega = 0.75$		
R&D-int.	0.94	0.96	0.08	0.09	0.97	0.14	0.15
Small	1.39	1.18	0.00	0.00	1.04	0.00	0.00
Young	2.09	1.80	0.00	0.01	1.61	0.00	0.02
Gazelles	2.68	2.39	0.02	0.03	2.20	0.03	0.05
All	1.00	1.00	0.15	0.00	1.00	0.26	0.00

The table presents the Bang for the Buck and its components when we account for technological spillovers between firms. We use patent values estimated in Kogan et al. (2017) to measure R&D quality. See also notes to Table 8.